Saying and Showing in Frege and Wittgenstein

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> The Lord who gives oracles at
> Delphi neither tells nor hides, but
> shows. — Herakleitos.

One of the most characteristic, notorious, and difficult doctrines of the *Tractatus* is the doctrine of what shows forth *(zeigt sich)* but cannot be said. Wittgenstein holds that various features of reality come out, *sich zeigen,* in our language, but we cannot use this language to *say,* assert, that reality has these features: if we try to frame propositions ascribing these features to reality, then it will be possible to show that strictly speaking these are not propositions, only sentence-like structures which violate the principle of logical syntax and are thus devoid of any sense, true or false. All the same, these nonsensical *(unsinnig)* structures may be useful; they may serve to convey from speaker to hearer an insight that cannot be put into proper propositions.

Readers of the *Tractatus* may easily come to think that this doctrine, even if it is not an elaborate mystification with something of charlatanism about it, is at best self-frustrating. If it follows from a philosophical doctrine that that doctrine cannot be intelligibly stated, surely that is a decisive refutation of the doctrine? I myself once thought so; it seemed to me that the predicament is one not peculiar to Wittgenstein's philosophy, but all the same, in the style of chess manuals, I spoke of *Ludwig's self-mate.* Such a self-mate is to be found, I thought, in the neo-scholastic doctrine that existence, *l'être,* cannot be conceptualized and can figure in our thought only by way of judgment asserting existence: but this very doctrine, I protested, conceptualizes existence, and in judging it to be true people are not judging that so-and-so exists, so it refutes itself. For *this* doctrine there is indeed nothing to be said: it is sufficiently refuted by other considerations — e.g. that an existential proposition, like any other,

need by no means express any *judgment,* since it can occur unasserted as a part of an assertion. But the charge of self-mating was a shallow reason for rejecting it.

I have come, I hope, to a better understanding of the matter through reflection upon 'the great works of Frege': such reflections can never be out of place for anybody who seriously wants to understand Wittgenstein — including Wittgenstein's later works. The influence of Frege on Wittgenstein was pervasive and life-long, and it is not of course just confined to places where Frege is mentioned by name or overtly referred to (cf. *Zettel* 712). I shall here argue that some fundamental aspects of the Wittgensteinian saying/showing contrast are already to be discerned in Frege's writings; there, of course, it is applied only in the philosophy of logic, and there, I shall argue, it is very strongly defended and defensible — it is hard to set up any coherent alternative. Wittgenstein extended the doctrine beyond the limits within which Frege employed it; this extension is one that cannot be knocked down at once if the unextended Fregean doctrines will stand up; but we shall see, I think, that it involves peculiar difficulties which cannot be readily overcome. Since I am not persuaded myself that this part of the doctrine is sound, I shall confine myself to expounding both the doctrine and the obvious objections; I must leave any serious defence to others.

I shall maintain in this paper the following theses:

(1) Frege already held, and his philosophy of logic would oblige him to hold, that there are logical category-distinctions which will clearly show themselves in a well-constructed formalized language, but which cannot properly be asserted in language: the sentences in which we seek to convey them in the vernacular are logically improper and admit of no translation into well-formed formulas of symbolic logic. All the same, there is a test for these sentences' having conveyed the intended distinctions — namely, that by their aid mastery of the formalized language is attainable.

(2) The category-distinctions in question are features both of verbal expressions and also of the reality our language is describing; in consequence, the manoeuvre of 'semantic ascent' — transformation of talk about things in the world into talk about expressions in a language — is in principle entirely futile as an attempt to resolve problems, or in particular to remove the difficulty about unsayables raised under (1).

(3) Wittgenstein's *Tractatus* accepts thesis (1) from Frege, modified only by the recognition of further category-distinctions which Frege did not recognize; thesis (2) is merely taken over, and stated with greater emphasis.

(4) The Frege-Wittgenstein notion of what comes out but cannot be asserted is almost irresistible, in spite of its paradoxical nature, when we reflect upon logic. Wittgenstein's view is that superficially indicative sentences of ethics, aesthetics, and religion must be assigned the same role as the strictly improper sentences used didactically in logic: the role of conveying insights. The difficulty besetting this further view is that it is a much more obscure question how we can tell in these cases that an insight actually has been conveyed.

(1) Before discussing the genuine difficulties over Frege's category-distinctions, it will be well to sweep away some mare's nests: to be sure, he himself constructed the mare's nests, but he also, as his *Nachlass* reveals, discovered how to demolish them. – Consider such phrases as:

(a) The reference *(Bedeutung)* of the predicate '– killed Caesar'.
(b) The reference *(Bedeutung)* of the functor 'the square of –'.
Frege would of course say that the predicate stands for, *bedeutet*, a concept, and that the functor *bedeutet* a function. Should we then get significant and true sentences by attaching to phrase (a) the predicate 'is a concept', and to phrase (b), the predicate 'is a function'? No, Frege would say, that cannot be right: phrase (a) is plainly not itself a predicate, nor is phrase (b) a functor; so phrase (a) cannot stand for a predicate, nor phrase (b) for a function, as functors do. By their syntactical structure, both (a) and (b) are definite descriptions, *Eigennamen* in Frege's sense, and must therefore stand for objects!

This is the corner that Frege had painted himself into when he wrote 'Über Begriff und Gegenstand'. Many people have thought he could not possibly extricate himself; some have even on the contrary supposed that Frege deliberately adopted an 'ontology' including special classes of objects that were surrogates for concepts and functions in case of need! But Frege could and did find a way out. The result of inserting an English expression in the blank between the quotes in the context:

what '   ' stands for

will stand for, *bedeuten,* whatever that very English expression stands

for; and similarly the result of inserting a French expression into:

what '   ' stands for in French

will stand for whatever the English translation of that French expression stands for. Thus the expression:

what 'the Duke of Wellington' stands for

is merely a long-winded substitute for the Duke's name; but on the contrary in the sentence

He is what 'un coquin' stands for in French

the predicate stands for what this French expression does stand for, and the sentence is a long-winded way of saying 'He is a scoundrel'. Similarly

that function of 2 which 'the square of' stands for

is a long-winded way of saying 'the square of 2': it would be nonsense, Frege came to see, to try to use the expression corresponding to (b) above:

that function which 'the square of' stands for

on its own as an *Eigenname,* or to try to use phrase (a) above as an *Eigenname* either.

Frege thus escapes checkmate at this move: but at the next move there will be a new check not so easily escaped. Frege holds that there is a fundamental difference between concepts and objects. Let us take an instance of this:

(c) There is a difference between what 'Brutus' stands for and what the predicate '– killed Caesar' stands for.

By the principle laid down above, any phrase of the structure:

what '   ' stands for

is replaceable syntactically by simple use of the English expression standing between the quotes – syntactically and indeed *salva veritate*. Let us try to do this with (c)! What we get is:

(d) There is a difference between Brutus and killed Caesar.

And this is manifest nonsense – and it would not make any improvement if we inserted some indication of an unfilled place for a subject before 'killed'!

Is Frege then checkmated after all? I do not think so. What he must

say, and I think would say, is to this effect: 'The reduction of (c) to (d) shows that we cannot really construct any significant proposition to say the sort of thing we were trying to say in (c). And this is not due to some removable defect of ordinary language: in a proper symbolic language, such as my *Begriffsschrift,* an inequality with an *Eigenname* on one side and a predicate or functor standing alone without its argument on the other side would equally not convey what we wish. All the same, sentences like (c) are didactically useful: they may lead someone to understand my *Begriffsschrift*. And the test of his having actually mastered the symbolic language is his successful use of it, not his ability to parrot sentences like (c) or produce similar ones on his own account.'

It is worth while to see why a symbolic language is equally inept to capture the sort of thing we are trying to say in English quasisentences like (c). Suppose we try to symbolize in a concrete instance the difference between a number and a function: say, $2 \neq \log_{10}$. The trouble is not that this must be nonsense; it already has sense in our symbolism, only the wrong kind of sense. '$2 \neq \log_{10}$' is an incomplete expression, a predicate, which turns into a mathematical sentence if we write some numerical expression after it; '$2 \neq \log_{10}10$' is true, and '$2 \neq \log_{10}100$' is false. The kind of sense '$2 \neq \log_{10}$' already has excludes it from being used to say that the number 2 is non-identical with a certain function; and any other attempt to assert such category-differences in a well-constructed symbolism must equally fail.

(2) A solution that is often offered to the difficulty just mentioned is that it comes about from trying to discuss in the object language what ought properly to be discussed in a metalanguage. '$2 \neq \log_{10}$' will indeed not bring out the difference we are trying to convey; but is it not obvious on reflection that the signs '2' and '$\log_{10}$' are not merely different signs but differently signifying signs? Frege's trouble would be diagnosed as his having thought of all signs as *names* — a diagnosis confirmed by his use of the term *'Functionsnamen'* for functors. If all signs have to be names, each one standing for something, then signs that do not name or stand for objects will have to be assigned some strange non-objects, such as concepts and functions, as their *Bedeutungen*. What he ought to have done is to distinguish the different mode of significance of signs; instead he misconstrued these distinctions as differences of ontological category between things named.

The diagnosis may well be suspected because Frege did not think

all signs in his symbolism must be assigned *Bedeutung:* not the assertion sign, nor yet any sign formed with it by prefixing it to another sign; and not variables either — indeed Frege used the purely syntactical expression 'letter' to avoid the very suggestion of such entities as variable magnitudes or the like *in rebus*. And here Frege did introduce three different semantical terms: a formula beginning with the assertion sign *behauptet* (asserts), a proper name *bedeutet,* a variable *deutet etwas unbestimmt an* — indefinitely indicates something. So Frege was perfectly capable of distinguishing different semantical roles: he was not stuck once for all with the idea of *Bedeutung*.

But the diagnosis fails more radically than this. It fails because, unlike Frege, the objectors stick to an ill-chosen, narrow, range of functional expressions. The piece of type '$\log_{10}$' looks like a separately quotable expression for a function; and then it is plausible to say: What we need is to distinguish the roles of '100' and '$\log_{10}$', not to distinguish ontologically between numbers and functions. But consider two of Frege's own examples:

(e) $(2+ 3 \cdot 0^2) \cdot 0$; $(2+3 \cdot 1^2) \cdot 1$; $(2 + 3 \cdot 2^2) \cdot 2$
(f) $2 \cdot 1^3 + 1$; $2 \cdot 4^3 + 4$; $2 \cdot 5^3 + 5$

In (e) we have designations of values of a certain function for the arguments 0,1,2; in (f), designations of values of another function for the values 1,4,5. In neither case can we extract a separately displayable and quotable functor. Frege would indeed say that we had here two *Functionsnamen,* '$(2 + 3 \cdot \zeta^2) \cdot \zeta$' and '$2 \cdot \zeta^3 + \zeta$'; but these quoted expressions visibly do not occur as physical parts of the numerical symbols designating the values. The role of these expressions with a Greek letter xi in them is to serve as stencils for constructing numerical signs; the numerical signs themselves are values of a certain function *from* numerical expressions *to* numerical expressions; so the function/object distinction reappears at the level of language and language about language cannot be used to charm away the distinction.

A still more striking example is the identity function — that function whose value always just is its own argument. Frege says *(Grundgesetze,* vol. i, p. 43) that the *Functionsname* for this function is simply 'ζ'; here there is no separate bit of print that anybody might take as playing role of a functor; but the rule for obtaining a designation of a value from one of an argument is still the same as in other cases — write down the name of the argument wherever there is a xi in the stencil. So the identity function *in rebus* is represented by an identity

function of designations. There could not be a more complete refutation of the 'explanation' that Frege erred by forgetting that different bits of type may have different semantical roles; and *this* use of the term *'Functionsname'* shows how far he was from being misled by the thought of a *name*.

In fact, the Fregean *Functionsnamen* can *never* figure as physical parts in Frege's logical notation, since the Greek consonantal letters never occur in any formula. Of course such a sign as 'log₁₀' could appear (if suitably defined); but this is not the *Functionsname* − 'log₁₀' is. And the point of insisting on this is that an isolated functional sign is for Frege a monstrosity; what signifies a given function is not the presence in a formula of a given piece of type, but the occurrence of a given pattern − in this case, the prefixing of 'log₁₀' to some proper name or some proxy for a proper name.

The same goes for predicates, and for symbolic concept-expressions: the occurrence of a predicate too must be recognized from the occurrence of a pattern, not from the occurrence of a quotable part of a sentence. Even when one is tempted to identify a predicate with such a quotable part, the presence of the predicate is constituted not by this but (say) by this quotable part's being appended to a proper name, or some proxy for a name such as a relative pronoun. Frege slightly muddied the water here by a footnote to 'Über Begriff und Gegenstand', in which he irrelevantly mentioned the fact that in the sentence:

The grammatical predicate 'is red' belongs to the subject 'this rose'

− supposed to relate to the sentence 'This rose is red' − the phrase:

The grammatical predicate 'is red'

is itself a grammatical subject. What *is* relevant is that the phrase 'is red' is not identifiable as a *logical* predicate; even in this simple case we have to think of a stencil 'is red' and a procedure for constructing actual sentences from this − which brings us back to the business of argument and function. And in more complex cases this is even obvious: the various sentences obtained by filling up all three blanks the same way in the stencil:

Nobody but − is sorry for − as much as − is

are clearly saying the same thing about the persons whose names successively fill the blanks, and may logically be regarded as having a common predicate, but this cannot be identified with any separately quotable and displayable part of the sentences.

Everything speaks, then, for the use of the argument-and-function apparatus on the linguistic level as well, and against any attempt to evade it by 'semantic ascent'. But should we speak of sentences that share a common predicate, with different subjects, as possessing a common property (that is, in Fregean jargon, as falling under our concept) or rather as values of one function for different arguments? This seems to matter little: and either description seems to fit equally well a series of complex designations of values for one and the same function, say:

(g) $2 \cdot 3^2 + 3, 2 \cdot 5^2 + 5, 2 \cdot 1000^2 + 1000$

and a series of predications saying one and the same thing about different objects, say:

(h) $2 \cdot 3^2 > (20 + 3), 2 \cdot 5^2 > (20 + 5), 2 \cdot 1000^2 > (20 + 1000)$

Here, I think, we find the real ground for Frege's decision to treat concepts as a special case of functions, sentences as a special case of names. Symbolic conveniences would have weighed little with Frege: the formal changes that would be needed to unscramble the mixing-up in *Grundgesetze* of the two categories of name and sentence would in fact be comparatively slight − when people nowadays refer to Frege's logic and set theory they often make the required adjustments tacitly. In fact there is very little change in the *look* of formulas when we pass to *Grundgesetze* from *Begriffsschrift*, although the categories of name and sentence are disjoint in one and pooled in the other. In any event, Frege was never the man to put symbolic convenience before truth.

What did clearly weigh with Frege was that numerical expressions for values of a given function from numbers to numbers, on the one hand, and propositions about numbers that fall under the same concept, on the other hand − for example, (g) and (h) above, respectively − could so well be treated in the same way, as values of a single *linguistic* function from names to complex expressions. Frege was thus led to regard sentences and complex numerical expressions as being alike names, *Eigennamen;* of the values of *nonlinguistic* functions. Once this step had been taken, Frege came to the question what objects sentences are names of; for reasons we need not here go into, his decision was − leaving aside quoted and *oratio obliqua* occurrences of sentences − that sentences name truth-values: more exactly: all true sentences have a common reference, which we may call

'the True'; all false ones likewise have one, which we may call 'the False'.

Some criticism of this miss the target. Frege is *not* confusing the semantical roles of stating and naming; he expressly distinguished these roles. It is on the contrary his critics who are confused when they say that the semantical role of 'making a statement' must be fulfilled if a sentence is to be adjudged true or false; for there are certainly some truth-functionally complex sentences for which the truth-value of the whole sentence, which perhaps is asserted, depends on the truth-values of *unasserted* sub-sentences. What can fairly be said is that Frege gives a most implausible account of what sort of performance assertion is. Apart from assertion, one is merely uttering a complex name of one or other object, the True or the False; assertion would presumably be a warranty of the name uttered *as* a name of the True. But assertion is an intentional act; so someone who asserts must *intend* to guarantee his sentence as a name of the True, and presumably in order to have this intention must know or believe beforehand that the sentence names either the True or the False! This appears to have been what Frege actually held; for he writes 'These two objects are recognised, if only implicitly, by everybody who judges something to be true — and so even by a sceptic.'

A trouble then arises over dialectical reasoning in Aristotle's sense — reasoning from unasserted premises, assumed, as we say, for the sake of argument. Aristotle already said that the formal rules of logic can take no account of the provenance of premises, and must on the contrary be just the same for reasoning from the asserted premises of some science and for dialectical reasoning. Frege absolutely denies the existence of dialectical reasoning; all premises must be asserted premises — dialectical reasoning 'Let us suppose that P; in that case, Q' is just a misleading way of presenting a logical assertion 'If P then Q'. Frege's assertion sign is in fact a valuable aid in practice, in the analysis of ordinary-language argument for keeping tabs on which premises are asserted and which are only dialectically assumed; over this there may easily arise confusion (say) as to whether someone is offering a valid *reductio ad absurdum*, or has reduced himself to absurdity by *asserting* a premise and then deriving from this and the other premises the contradictory assertion as a conclusion. But Frege's own mistaken view that sentences are names ironically estopped him from recommending the use of his assertion sign in this connexion: without the warranty expressed by the assertion sign, a sentence is

for Frege a mere name, which can no more figure as premise or conclusion than 'Julius Caesar' can, or an astrological sign for a planet, or a numeral. — It is even more ironical that in the Appendix on Russell's paradox Frege actually gives *two* derivations of the paradox in his own symbols, with the inferences drawn according to his own formal rules, but with assertion signs omitted throughout — *'wegen der zweifelhaften Wahrheit'*, as he says, because the truth of the matter is doubtful. (*Grundgesetze* vol. ii, pp. 256–257.) So Frege himself (driven by the very truth, as Aristotle would say) reasoned dialectically in this place from unasserted premises just as if he had been reasoning from asserted ones.

In the *Tractatus* Wittgenstein strongly asserted the Aristotelian view of this matter against Frege; but he over-reacted by declaring the assertion sign to have only psychological, not logical, import. Clearly it is a *logical* matter whether someone is deriving a conclusion from asserted premises and with this good logical warrant going on to assert it, or on the contrary contradicting himself; and just this is the difference — supposing that 'P, Q, R' is indeed a premise set logically yielding 'not R' — between the valid *reductio ad absurdum:*

P, Q; suppose that R; then not R; *ergo* not R

and the self-reduction of the arguer to absurdity:

P, Q, R; ergo not R.

(The capital letters are here not variables for which substitutions may be made, but are meant to be short for suitable actual sentences as examples, which I leave it to the reader to supply.) Here the assertion sign plays an essential logical role; the correct placement of the sign will discriminate between the two cases, valid argument and self-contradiction, in such a case as an epistemological argument that uses the contents of some memory as a premise in order to prove that memories are not always reliable.

A further fault in Frege's treatment of sentences was that it led to a blurring of his valuable insight (adopted into the *Tractatus*) that a name's *Bedeutung* depends on the possibility of embedding it in a sentence. If sentences are names, then this insight will be transmogrified into the following: A name that is *not* a name of the True or the False owes its *Bedeutung* to the possibility of making it into part of a longer name that *is* a name of one of these two objects! And who would ever maintain this, except a theorist committed to maintaining the consequences of his theory however implausible?

(3) I think a great deal of the *Tractatus* is best understood as a re-fashioning of Frege's function-and-argument analysis in order to remove this mistaken treatment of sentences as complex names. For Wittgenstein, just as for Frege, a sentence will be a certain function of the names that occur in the sentence, and what corresponds *in rebus* to the sentence will be a certain function of the objects so mentioned; but now neither is the sentence a complex name, nor is what corresponds to it *in rebus* a named object, the True or the False; the sentence is a *fact,* and is true or false in virtue of its according or discording with *facts* about the things mentioned.

Wittgenstein speaks of the way that a relation between things named is represented by a *relation* between their names in the sentence affirming that the relation holds (3.1432); but he might equally well, or better, have spoken of functions. The fact that *a* is on top of *b,* and the fact that *c* is on top of *d,* are two values of one and the same function, first for the pair of arguments *a, b,* and then for the pair of arguments *c, d;* on the level of language, the sentences '*a* is on top of *b*', '*c* is on top of *d*' are themselves facts about the pair of names '*a*', '*b*' and the pair of names '*c*', '*d*' respectively, and the first linguistic fact is the same function of the pair '*a*', '*b*' as the second is of the pair '*c*', '*d*'. In regard to this illustration, I am of course open to the protest that I have not given examples answering to Wittgenstein's requirements for *atomic* facts. But neither Wittgenstein nor anybody else has produced an example of a sentence affirming that there obtains a fact which would count as atomic. For reasons that will appear later, I regard the notion of an atomic fact as one reached only through a serious confusion on Wittgenstein's part (which he later acknowledged) and incompatible with other more defensible theses of the *Tractatus* theory.

Frege was led to assimilate complex names and sentences because the *linguistic* functions forming these complex expressions appeared similar. Wittgenstein turned this assimilation upside down. For Wittgenstein, there were no name-forming functions operating upon names at the linguistic level, and no functions from objects to objects on the level of what is signified. 'A complex sign', he wrote in a notebook, 'just *means*' 'a sentence': all complexity of signs is the complexity of sentence-structure − and a sentence is not a name but a fact. Apparent complex names are to be eliminated by Russell's Theory of Description; all genuine names are *syntactically* simple − their physical complexity is always irrelevant to their use as names, be-

cause there is no *particular* kind of physical complexity that a name needs in order to do its naming job (3.261, 3.3411). With the elimination of complex names, name-forming functions with names as arguments disappear: we have in all cases, at the linguistic level, *proposition*-forming functions with names as arguments (propositional functions, in *one* of Russell's several uses of that term).

We should here notice that in the *Tractatus* view of the matter a phrase like 'the fact that the cup is on top of the saucer' is *not* a definite description of a fact, exponible in some such form as 'the one and only fact in which the relation *on top of* binds the cup to the saucer': a description which would become vacuous if this relation does not really bind the cup to the saucer, and which anyhow ought to be eliminated by Russell's Theory. Such a view is false to the *Tractatus* theory in several ways. Wittgenstein, as is clearly shown in the Notebooks, would no more tolerate a relation's being presented by a free-standing, namelike expression like 'the relation *on top of*' than Frege would have; for him as for Frege such an isolated functor was a monstrosity − 'on top of' would be significant only in a context where its two argument-places were filled with names or name-variables. Moreover, the grammatical similarity to a definite description is misleading: '. . . a description of an object describes it by giving its external properties, . . . a proposition describes the reality by the internal properties of the reality' (*'ihren internen Eigenschaften'*, where *'ihren'* must look back to *'die Wirklichkeit'* − 4.023). And the formal or internal or structural properties of the reality of the fact that obtains *in rebus,* are not like the properties used to frame the description of an object: for properties are internal when *there is no such thought* as the thought that they do not apply to what has them (4.123), and we cannot use a proposition to ascribe or deny the presence of a formal property (4.124). So this way of contruing 'the fact that . . .' phrases is ruled out (contrary to a view Moore once maintained). Wittgenstein would no doubt *not* object if for sentences containing such phrases we gave an analysis which destroyed the phrases' name-like appearance; but the sentence-structure following 'the fact that' would then *not* be dissolved, as it would on the definite-description account.

We can do something to elucidate this rather perplexing talk about internal properties. The common pattern of the facts that *a* is on top of *b* and that *c* is on top of *d,* which makes them to be values of the same function for different arguments, is a place where we have to stop − rock-bottom, where the spade turns. We cannot analyse away

these functions, as we can the functions Russell called descriptive functions (e.g. *the father of, the square of*), in terms of a common relation to different objects; the holding of such a common relation would bring us back again to another and more complicated instance of the very thing we were trying to explain. In *The Principles of Mathematics* Russell has this very argument concerning propositional functions (§ 82); what he *here* means by this very ambiguous term is: functions *from* objects *to* propositions whose constituents are the objects themselves, not *verbal* propositions *about* the objects. The proposition so conceived is not the same as a Wittgensteinian fact; but it seems clear that the argument carries over, and likely that Wittgenstein himself was led this way by his reading of Russell.

Functions from names to names are abolished by Wittgenstein's reform of the Fregean theory: but there still are in Wittgenstein functions from sentences to sentences, and sentences are a kind of facts. These functions are sometimes called operations, sometimes truth--functions, but never functions *simpliciter* — a term he reserves for functions that form propositions from names. Physically, prefixing a negation-sign (say) looks much like prefixing a predicate-letter; but the way of signifying is quite different. Negation, for example, can be iterated, and then cancels itself out; no value of a propositional function can be argument of that function — '(Socrates smoked) smoked' is uninterpretable, because' smoked, is only a receipt for getting sentences from *names* (cf. 5.251). We should better see how the truth functions are fact-forming operators upon facts if we used a notation suggested by Ramsey, in which a proposition is negated by rotating its sign around a horizontal axis. (A hemicylindrical lens would enable one to read off negations, if laid on the page: for names and name-variables, which must occur alike in a proposition and its negation, one would use such signs as the letters of CHOICE BOX which are unchanged by such rotation.) Then it is clearly visible how the sentence-facts that are negations of one another are each derived from the other, and how double negation cancels out.

The parts of the *Tractatus* theory that I have here sketched must be regarded as an attempt by Wittgenstein to accept Frege's theses (1) and (2) — particularly the parallelism between category-distinction *in rebus* and in the domain of language — without incurring the implausibilities of the doctrine that sentences are names. A further instance of this parallelism of categories, and of the consequent futility of semantic ascent as a solution for philosophical problems, is men-

tioned by my Warsaw friend Boguslaw Wolniewicz in his book *Rzeczy i Fakty (Things and Facts:* Warsaw, 1968). In the *Tractatus* Wittgenstein finds it necessary to bring in possible facts, possible states of affairs, which could exist but do not. It has been the way of positivist philosophers to treat modal attributes like possibility as attributes of *sentences,* and then as a matter of language about language, with no metaphysical taint. But of course logical semantics requires that there are possible sentences too — not in the sense of ones that would be said by non-positivists to *affirm* something possible, but in the sense of sentences that might be framed but never have been; and these in turn are simply possible facts (facts about the putting-together of words) that might have obtained but never did — so the problem of the possible and the actual remains just where it was before.

There are less satisfactory aspects of *Tractatus* doctrine, not to my mind logically implied by those so far discussed. For example, Wittgenstein held that the objects named must be simple. This doctrine altogether wrecks the doctrine of a parallelism in logical structure between sentence-facts and the facts they express *in rebus;* for names, however irrelevant semantically the inner complexity of a name may be, certainly are not simple objects. Wittgenstein regards a complex 'object' as consisting in the *fact* that certain objects, parts of the complex, are related thus and so; if this holds, then facts about names will always have a radically different structure from facts about simple named objects — not to add that we have no names for simple objects anyhow. Again, the objects with regard to which sentence-facts are facts cannot be just token-words, they must be type-words; it is fairly clear that without this there are many place where the theory would not work; anyhow, Wittgenstein expressly says at 3.203 that '*A*' is the same signs as '*A*'. But type-words are not Wittgensteinian simple objects. Wittgenstein is right, I think, in his argument for the substance of the world, the objects: there cannot be facts about facts . . . ad infinitum: we must in the end come to objects that are not facts, which facts are directly or indirectly about. McTaggart argued for the existence of 'substances' in essentially the same way. But Wittgenstein confused a fact about *A* and *B* with a complex containing *A* and *B* — a confusion due to Russell (from which incidentally McTaggart was quite free). Russell had used expressions like 'knife-to-left-of-book', which confused together a *whole* having knife and book as its parts and the *fact* that the knife is to the left of the book; he expressed this confused notion with the term 'complex'. Wittgenstein told me once

that after the *Tractatus* was published Frege asked him whether a fact is *bigger* than each thing it is about: this criticism made no impression at the time, but eventually led him to abandon the notion of facts as complexes. But once this confusion is cleared up, there is no reason why the objects, the substances, should be simple (McTaggert in fact held that *every* substance has parts *ad infinitum);* and then the parallelism of logical structures in language and the world can be restituted — and what we might ordinarily take to be namable may really be such, and so may names themselves be objects.

The other big divergence between Frege and Wittgenstein that I have not so far discussed is on the question whether numbers are objects. There is very little I can say about this. Frege's reason for treating numbers as objects namable by proper names is the *profound* grammatical similarity between talk about numbers and talk about the sort of objects to which we give ordinary proper names. What logically behaves in every way as a proper name *is* a proper name, so on the face of it numerals are proper names. The *Tractatus* in no wise convincingly demonstrates the breakdown of this grammatical similarity from a logical point of view; nor is the very sketchy and fragmentary account of mathematical expressions remotely considerable as a rival to Frege's work, not even to the *Grundlagen* by itself.

The indescribability of logical structure, the structure common to linguistic sentence-facts and the facts in the world that they are meant to represent, is of course a *Leitmotiv* of the *Tractatus*. With the Fregean background filled in, the doctrine should no longer appear as mystification, even if it remains mysterious: and the difference between sentences and names, like that between names and predicates in Frege, will be seen not as a matter of intuitively discriminating the strings of signs that make sense and the others that are nonsense, but as coming out whenever we grasp the roles of *significant* strings a language — and there will be publicly available tests (including, perhaps, examinations!) for whether this grasp has been achieved.

(4) Paradoxical as is the doctrine of aspects of reality that come out but cannot be propositionally expressed, it is hard to see any viable alternative to it so long as we confine ourselves to philosophy of logic: and in this domain Wittgenstein revised Frege's views without unfaithfulness to Frege's spirit. The category-difference between sentences and names was upheld by Frege himself when he wrote *Begriffsschrift;* the naming role of numerals was something about which he said to Wittgenstein 'Sometimes I seem to see a difficulty, but then

again I don't see it' — and at the very end of his life he was looking for some alternative account of numerals.

Wittgenstein however held, not only that the fundamental category-distinctions of logic can be discerned but not expressed in properly intelligible well-formed propositions, but also that the same holds for the insights conveyed through ethical, aesthetic, and religious utterances. He may well have believed that the failure of these utterances to be genuinely propositional could be demonstrated by what he calls *the* correct philosophical method (6.53: cf. 5.4733); namely, if we try to construe them as propositions, we shall always find a failure to give any *Bedeutung* to some sign employed. But he gives no typical instance of such a religious, aesthetic, or moral utterance, with the failure to assign *Bedeutung* clearly established.

Wittgenstein's reference to this breakdown of meaning relations is rather reminiscent of an episode in H. G. Wells' grim science-fiction story, *The Island of Dr Moreau.* An ape that has been surgically given the power of speech distinguished between Big Thinks and Little Thinks: Little Thinks were concerned with the affairs of daily life, Big Thinks were about 'names that meant nothing', and the ape thought it made him into a true man that he could engage in Big Thinks. On his return from captivity, the narrator found that it seemed to him as he turned aside into some chapel 'that the preacher gibbered Big Thinks even as the Ape Man had done'. Some positivists have certainly come close to regarding as Big Thinks the whole class of the sentences of ethics, aesthetics, and theology.

In spite of the verbal similarity between Wells' mention of 'names that meant nothing' and Wittgenstein's talk of failing to assign *Bedeutung,* it would diametrically misrepresent his attitude towards moral, and aesthetic and religious utterances to use the term 'Big Thinks'. Obviously he held that in such discourses elements of Big Thinks can be found; he strongly disliked a once familiar style of would-be rational apologetic from Catholic priests, and from when he first read it he regarded Moore's *Principia Ethica* as radically confused. But *some* moral, aesthetic, and religious utterances would serve the same sort of purpose as do in logic the elucidatory sentences that introduce us to the use of logical notation: namely, none of these sentences are both syntactically well-formed and semantically supplied with *Bedeutungen* for the expressions employed, but they may nevertheless succeed in conveying insights.

In view of the parallel with the logical case, the position cannot be

contemptuously dismissed; but it rasises a special difficulty of its own. The insight we gain, by aid of strictly nonsignificant elucidatory sentences, into the workings of logical notation can be definitely tested — even by University examiners. But what is to be the test that ethical, aesthetic, or religious sentences have similarly conveyed a genuine insight? So far as I can tell, no answer is to be found in anything Wittgenstein wrote.

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The Sentence as a Function of its Constituents in Frege and in the Tractatus

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In proposition number 3.318 of the *Tractatus* Wittgenstein says: "Like Frege and Russell I conceive of a sentence as a function of the expressions contained in it."

We see from this statement that Wittgenstein thought there was some kind of similarity between his conception of a sentence and that of Frege's or Russell's. But of course it does not mean that he took a sentence to be a function in the same sense as either of these two philosophers. I shall in this paper point out some similarities and differences between Wittgenstein's conception and Frege's conception (as it is reflected in the papers mentioned below).

I shall first give a statement of Frege's view (sections 1–12) and then proceed to Wittgenstein (sections 13–21).

1. In his famous paper "Function and Concept", to which Wittgenstein seems to refer in different places in the *Tractatus,* Frege takes his starting-point from the question: What is called a function in mathematics? and he says that mathematicians were likely to answer that a function of x is a "mathematical expression[1] containing x, a formula containing the letter x" ([1], p. 21; [2], p. 18). So, for instance, one would say that

(1)   The expression "$2 \cdot x^3 + x$" is a function of x
and
(2)   The expression "$2 \cdot 2^3 + 2$" is a function of 2.

Now this formulation did not, I think, conform with anything that would have been regarded as mathematical common sense at Frege's time; it is rather an exaggerated statement of a formalistic standpoint in the philosophy of mathematics. And since what is important in Frege's argument refers to the definition