Philosophy 230

Wesleyan University
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Handout 8a

Deductions, II

I. Informal arguments.

A. "Something is F" implies "Something is either F or G."

1. Something is F.
2. Let us call it Roscoe.
3. So Roscoe is F.
4. So Roscoe is either F or G.
5. So something is either F or G.

B. "Someone is not President" implies "The moon is made of green cheese"

1. Someone is not President.
2. Let us call her Barak Obama.
3. So Barak Obama is not President.
4. But Barak Obama is President.
5. So Barak Obama is both President and not President.
6. So, the moon is made of green cheese.

II. The Rules of Existential Instantiation:

Rule *EII* (Existential Instantiation Introduction):

> Suppose that on some line (m) we have an existentially quantified schema. Then on any subsequent line (n) we may put a *conservative* instance of that schema, with premise numbers those of line (m) plus (n) itself. The citation is "$(m)u$", where "u" is the instantial variable. The variable "u" is said to be *flagged* at this line.

Rule *EII* (Existential Instantiation Elimination):

> Suppose that on some line (m) there is, among the premises, line (j), which was obtained by EII, and that the variable flagged at line (j) does not occur free in line (m) *nor in any premise of line* (m) *other than* (j). Then on any subsequent line we may put the same schema as occurs on line (m), with premise numbers those of line (m) except for (j). The citation is $[j](m)$.

III. *EI* Deductions

A. The formalization of the informal argument above: deduce "$(\exists x)(Fx \vee Gx)$" from "$(\exists x)Fx$".

B. Show that "$(\forall x)(Fx \supset Gx)$" and "$(\exists x)Fx$" imply "$(\exists x)Gx$".

C. Show that "Fa" and "$(\exists x)Gx$" imply "$(\exists x)(Fx.Gx)$".

D. Show that the following argument is valid:

Premises

If anyone spoke to anyone, then someone introduced them.

No one introduced anyone to anyone unless she knows them both.

Everyone spoke to someone.

Conclusion

THEREFORE, everyone knows someone.

IV. Liberalized UG

A. The rationale for this rule is to save some steps in deduction. Consider the following deduction:

[1]	(1)	$(\forall x)(\forall y)Gxy$	P
[1]	(2)	Gxx	$(1)MUI$
[1]	(3)	$(\forall x)Gxx$	$(2)UG$
[1]	(4)	Gyy	$(3)UI$
[1]	(5)	$(\forall y)Gyy$	$(4)UG$

Steps (4) and (5) seem like an unnecessary fandango; we instantiate a universal generalization only to generalize the instance, simply to get a different variable of quantification. Yet the rules we have forces us to go through this extra two steps.

B. So we extend the deduction system by allowing UG to a different variable of quantification. To be more precise, we first need the notion of *alphabetic variants.* $(\forall u)\Phi(u)$ and $(\forall v)\Phi(v)$ are alphabetic variants iff all free occurrences of u in $\Phi(u)$ are replaced by v to obtain $\Phi(v)$, and *vice versa.*

C. Now the rule of Liberalized UG, acronym LUG, allows us to infer from a schema $\Phi(u)$ any schema $(\forall v)\Phi(v)$, where $\Phi(u)$ and $\Phi(v)$ are alphabetic variants, provided that u is not free in any premises of $\Phi(u)$. The premise numbers and citations are the same as for UG.

D. With LUG we can shorten the above deduction to:

[1]	(1)	$(\forall x)(\forall y)Gxy$	P
[1]	(2)	Gxx	$(1)MUI$
[1]	(3)	$(\forall y)Gyy$	$(4)LUG$