Philosophy 230

Wesleyan University
Fall 2014

Handout 7a

Deductions, I

I. An argument for the validity of:

If every soprano snubs her understudy, then not every soprano fails to snub her understudy.

A. Suppose that every soprano snubs her understudy.

B. Then, say, Hildegard Behrens snubs her understudy.

C. But, then, some soprano snubs her understudy.

D. So, not every every soprano fails to snub her understudy.

E. Therefore, if every soprano snubs her understudy, then not every soprano fails to snub her understudy.

II. Rules of Deduction, Part I:

Rule *P* (Premise):

On any line (n), we may write any schema, with $[n]$ itself as the premise number.

Rule *UI* (Universal Instantiation):

Suppose that on line (m) we have a schema "$(\forall x)\Phi(x)$". Then on any subsequent line we may write any instance "$\Phi(y)$", with premise numbers those of line (m) and citation (m).

Rule *TF* (Truth Functional Implication):

Suppose that on lines (m_1), (m_2),..., (m_i) we have some schemata $R_1, \ldots, R_i$. Suppose further that $R_1, \ldots, R_i$ *truth-functionally* implies a schema S. Then on any subsequent line (n) we may write the schema S, putting as premise numbers all those of lines (m_1), (m_2),..., (m_i), citing all of these lines.

Rule *EG* (Existential Generalization):

Suppose that on line (m) we have a schema $\Phi(y)$ and that $\Phi(y)$ is an instance of "$(\exists x)\Phi(x)$". Then, on any subsequent line we may write $\Phi(y)$, with premise numbers those of line (m) and citation (m).

Rule *CQ* (Conversion of Quantifiers):

If on line (m) we have a schema of one of the forms of column 1. Then on any subsequent line we may put the corresponding schema in column 2, with premise-numbers the same as for line (m), and with citation (m):

1	2
$(\forall x) - \Phi(x)$	$-(\exists x)\Phi(x)$
$(\exists x) - \Phi(x)$	$-(\forall x)\Phi(x)$
$-(\forall x)\Phi(x)$	$(\exists x) - \Phi(x)$
$-(\exists x)\Phi(x)$	$(\forall x) - \Phi(x)$

Rule D (Discharge of premise):

Suppose that on line (m) we have a schema S and that one of the premise numbers of line (m) is k; suppose further that the schema on line (k) is R. Then, on any subsequent line we may put the conditional "$R \supset S$", with citation $[k](m)$, and with premise numbers all those of line (m) *except* for k.

III. An argument for the validity of

If every tenor is vain, then every tenor is vain or fat.

A. Suppose that every tenor is vain.

B. Consider now an arbitrary tenor, call him "x".

C. By supposition, whichever tenor x may be, he is vain.

D. So, x is either vain or fat.

E. But x was chosen arbitrarily; that is, we have supposed absolutely nothing about him except that he is a tenor.

F. So *every tenor* is either vain or fat.

G. Hence, if every tenor is vain, then every tenor is vain or fat.

IV. Rules of Deduction, Part II:

Rule UG (Universal Generalization):

Suppose that on line (m) we have a schema $\Phi(y)$ and that $\Phi(y)$ is a *conservative* instance of "$(\forall x)\Phi(x)$"; suppose further that "y" is not free in any premise on which line (m) depends. Then, on any subsequent line, we may write "$(\forall x)\Phi(x)$", with premise numbers those of line (m) and citation (m).

V. Establish by deduction:

A. "$(\forall x)(p \vee Fx) \supset [p \vee (\forall x)Fx]$" is valid.

B. "$(\forall x)[(\exists y)Fy \supset Gx]$" implies "$(\forall y)(\forall x)(Fy \supset Gx)$".

C. "$((\forall x)(p \supset Fx).p) \supset (\forall x)Fx$" is valid.

D. "$(\forall x)(Fx.(\forall y)Rxy \supset Gx)$" implies "$(\forall x)(Fx. - Gx \supset (\exists y) - Rxy)$".

VI. Multiple UI

We can shorten:

[1]	(1)	$(\forall x)(\forall y)(\forall z)(Fxy.Fyz \supset Fxz)$	P
[1]	(2)	$(\forall y)(\forall z)(Fxy.Fyz \supset Fxz)$	(1)UI
[1]	(3)	$(\forall z)(Fxy.Fyz \supset Fxz)$	(2)UI
[1]	(4)	$Fxy.Fyx \supset Fxx$	(3)UI

to

[1]	(1)	$(\forall x)(\forall y)(\forall z)(Fxy.Fyz \supset Fxz)$	P
[1]	(2)	$Fxy.Fyx \supset Fxx$	(1)MUI