Philosophy 290Philosophy 230

Wesleyan University
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Handout 6a

General Laws, II

I. Additional Rules

A. We have two other rules of this kind, which concern, not validity, but implications. The first is the **Law of Universal Generalization**:

> If a schema S implies a schema $\Phi(u)$ and S does not contain "u" free, then S implies "$(\forall u)\Phi(u)$"

B. The second is the **Law of Existential Implication**:

> If a schema $\Phi(u)$ implies a schema S and S does not contain "u" free, then "$(\exists u)\Phi(u)$" implies S.

C. The proofs of these laws are not difficult. Let us prove the first.

1. Suppose that S implies that $\Phi(u)$ and that S does not contain "u" free.
2. Consider an arbitrary interpretation I under which "$(\forall u)\Phi(u)$" is $\bot$.
3. There must be some object, in the UD of I, say, a, which, when assigned to "u", makes $\Phi(u)$ $\bot$.
4. Consider, however, the interpretation I', which is just like I, but which assigns the object a to "u".
5. Under I', then, $\Phi(u)$ is $\bot$; since S implies $\Phi(u)$, S must also be $\bot$ under I'.
6. But, since S does not contain "u" free, the assignment made to "u" does not affect the truth-value of S. Hence, S must also be $\bot$ under I.
7. But, since the interpretation I was arbitrary, S must be $\bot$ under any interpretation under which "$(\forall u)\Phi(u)$" is false. So, S implies "$(\forall u)\Phi(u)$".

D. For the second law, we proceed similarly. I shall leave the proof to you as an exercise.

II. The Relettering Law

A. We can generalize these laws by means of the **Relettering Law**:

> If $\Phi(v)$ is a substitution instance of $\Phi(u)$, then "$(\forall v)\Phi(v)$" is equivalent to "$(\forall u)\Phi(u)$" and "$(\exists v)\Phi(v)$" is equivalent to "$(\exists u)\Phi(u)$".

Clearly this holds just in case alphabetic variants are equivalent.

B. I prove just the part of this Law dealing with the universal quantifier.

1. Recall that $\Phi(v)$ is a substitution instance of $\Phi(u)$ only if $\Phi(v)$ contains "v" free exactly where $\Phi(u)$ contains "u" free.
2. This implies that $\Phi(u)$ does not contain free "v"; so "$(\forall u)\Phi(u)$" does not contain free "v".
3. Moreover, "$(\forall u)\Phi(u)$" implies $\Phi(v)$, since the latter is an instance of the former.
4. And, by UGI, therefore, "$(\forall u)\Phi(u)$" implies "$(\forall v)\Phi(v)$".

5. Conversely, "$(\forall v)\Phi(v)$" implies $\Phi(u)$; $\Phi(v)$ does not contain free "u", since all free occurrences of "u" are substituted with "v".
6. Hence, "$(\forall v)\Phi(v)$" does not contain free "u"; so, by *UG I*, again, "$(\forall v)\Phi(v)$" implies "$(\forall u)\Phi(u)$". Hence, they are equivalent.

III. The **Law of Deduction**, or the **Deduction Theorem**:

Schemata $R_1, \ldots, R_n$ imply schema S just in case $R_2, \ldots, R_n$ imply $R_1 \supset S$.