Philosophy 230

Wesleyan University
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Handout 4ab

Analysis of Polyadic Arguments;
Instances of Quantificational Schemata

I. Two Polyadic Arguments

A. Argument 1

1. Some logicians are philosophers
2. Some logicians respect no philosopher
3. THEREFORE, some logicians are not respected by all logicians.

B. Argument 2.

1. Any philosopher who isn't a logician is respected by a logician.
2. No logician respects all philosophers.
3. THEREFORE, a philosopher not respected by all logicians is a logician.

II. Notation for talking about **arbitrary** quantificational schemata.

A. I will from now on write "$\Phi(u)$", "$(\forall u)\Phi(u)$," and "$(\exists u)\Phi(u)$" to indicate arbitrary schemata with certain properties.

B. In this notation, "u" indicates an arbitrary, unspecified variable: for example,

1. "x"
2. "y"
3. "z"
4. "w", etc.

C. "$\Phi(u)$" indicates an arbitrary, unspecified, *open* schema, with "u" the unspecified free variable: for example,

1. "Fx"
2. "$Gy \supset Hy$"
3. "$Fz \equiv Hz \vee Gz$", etc.

D. "$(\forall u)\Phi(u)$," and "$(\exists u)\Phi(u)$" indicate arbitrary simple quantificational schemata, where the unspecified variable "u" is the same after the quantifier and in $\Phi(u)$, for example:

1. "$(\forall y)(Gy \supset Hy)$"
2. "$(\exists z)(Fz \equiv Hz \vee Gw)$", etc.

E. The last example shows that by writing "$\Phi(u)$" I indicate that *at least* "u" occurs as a free variable in "Φ", but there may be other free variable in "Φ" as well.

III. Substitution Instances

A. If we start with an open sentence "$\Phi(x)$," "$\Phi(y)$," called a **SUBSTITUTION INSTANCE** of $\Phi(x)$, is the open sentence that satisfies the two following requirements:

1. "$\Phi(y)$" results from "$\Phi(x)$" by replacing all **FREE** occurrences of "x" in "$\Phi(x)$" by "y"; i.e., substituting "y" for all **FREE** occurrences of "x".
2. No occurrence of "y" that results from this substitution is bound by a quantifier.

IV. Regular and Irregular Substitution Instances:

A. If a substitution instance contains more free occurrences of the new variable than the original schema contained of the original free variable, it is an **IRREGULAR SUBSTITUTION INSTANCE**.

B. If a substitution instance and original schemata contain the same number of occurrences of their respective free variables, then the instance is a **REGULAR SUBSTITUTION INSTANCE**.

V. Instances of quantified schemata, Conservative and Nonconservative:

A. "$\Phi(y)$" is an **INSTANCE** of "$(\forall x)\Phi(x)$" if "$\Phi(y)$" is a **SUBSTITUTION INSTANCE** of "$\Phi(x)$".

B. "$\Phi(y)$" is a **CONSERVATIVE INSTANCE** of "$(\forall x)\Phi(x)$" if $\Phi(y)$ is a **regular** substitution of $\Phi(x)$.

C. "$\Phi(y)$" is a **NON-CONSERVATIVE INSTANCE** of "$(\forall x)\Phi(x)$" if $\Phi(y)$ is a **irregular** substitution of $\Phi(x)$.

D. Similarly for "$\Phi(y)$" and "$(\exists x)\Phi(x)$".

VI. Alphabetic Variants

A. Suppose that "$\Phi(u)$" and "$\Phi(v)$" are substitution instances of one another.

B. Then "$(\forall u)\Phi(u)$" and "$(\forall v)\Phi(v)$" are **ALPHABETIC VARIANTS** of one another, as are "$(\exists u)\Phi(u)$" and "$(\exists u)\Phi(u)$".

VII. Test your understanding of Instances and Alphabetic Variants

A. Is $Fzz \supset (\exists y)Gzy$ an instance of $(\forall x)[Fxz \supset (\exists y)Gxy]$?

B. Consider the schema

$$(\exists x)[((\forall y)Fyy.(\exists z)Gzx) \vee (\exists w)Hxwt]$$

Are the following all instances? Which of the instances are conservative?

1. $((\forall y)Fyy.(\exists z)Gzs) \vee (\exists w)Hswt$
2. $((\forall y)Fyy.(\exists z)Gzt) \vee (\exists w)Htwt$
3. $((\forall y)Fyy.(\exists z)Gzz) \vee (\exists w)Hzwt$
4. $((\forall y)Fyy.(\exists z)Gzy) \vee (\exists w)Hywt$

C. Are $(\forall z)[Fzz \supset (\exists y)Gzy]$ and $(\forall x)[Fxz \supset (\exists y)Gxy]$ alphabetic variants?