Philosophy 230

Wesleyan University
Fall 2014

Handout 3b

Polyadic Schemata and Interpretation

I. Determining monadic validity and implication.

   A. Is $(\forall x)(Fx \supset Gx).(\exists x)(Fx) \supset (\exists x)(Gx)$ valid?

   B. Does $(\exists x)Fx.(\exists x)Gx$ imply $(\exists x)(Fx.Gx)$?

   C. Are $(\exists x)Fx$ and $(\exists x)(Fx \vee Gx).(\exists x)(Fx \vee -Gx)$ equivalent?

   D. Does $(\exists x)(Fx \vee Gx)$ imply $(\forall x)(-Fx) \supset (\exists x)Gx$?

   E. Is the following argument valid?

      1. If no spies are well-educated, then no well-educated person is deceitful.
      2. Wesleyan graduates are well-educated.
      3. THEREFORE, if any Wesleyan graduate is deceitful, some well educated people are spies.

II. A *polyadic interpretation* of a polyadic schema has four parts:

   A. A domain of quantification, specified by

      1. An English (one-place) predicate (replacement), or
      2. A non-empty set

   B. Interpretations of predicate letters, by

      1. English predicates (replacement), or
      2. One of the following sets:
         a. A subset of the domain, for one-place predicates,
         b. A set of ordered pairs of elements of the domain, for two-place predicates,
         c. A set of ordered triples of elements of the domain, for three-place predicates, etc . . . .
         d. In general, a set of ordered n-tuples, for each n-place predicate.

   C. Interpretations of free variables, by

      1. English names (replacement), or
      2. Elements of the domain

   D. Interpretations of sentence letters, by

      1. English sentences (replacement), or
      2. One of the two truth values, $\top$ or $\bot$.

   E. No variable bound a quantifier in the schema being interpreted becomes bound by another quantifier after the complex schematic or English predicate is substituted for the predicate letter.

III. Interpretation by Replacement: examples.

A. $(\forall x)((\exists y)Fxy \supset (\forall y)Fxy)$

1. The *universe of discourse* is interpreted by the extension of "① is a person".
2. "F①②"is interpreted by the English predicate: "① dislikes something about ②"

B. $(\forall x)(p \supset (\exists z)Fxz)$

1. The domain is interpreted by the extension of "① is a country".
2. The sentence letter "p" is interpreted by "The USA is in a recession".
3. "F①②" is interpreted by "① owes money to ②"

IV. Restrictions on the more general notion of interpretation by replacement:

No variable that is free in the schema being interpreted becomes bound after the complex schematic or English predicate is substituted for the predicate letter.

V. Interpretations by Assignment:

A. $(\forall x)((\exists y)(Fxy) \supset (\forall y)(Fxy))$

1. $DQ = \{$Dan Quayle, Llyod Bentsen$\}$
2. $extF = \{< DQ, LB >, < DQ, DQ >, < LB, DQ >\}$

B. $(\forall x)(Cx.(\exists y)(Px. - Lyx) \supset (\exists y)(Px.Lyx))$

C. $(\exists x)(Cx.(\forall y)(Cy \supset -Lyx))$

D. $(\exists x)(\forall y)(Fyx \supset Fyy)$

E. $(\forall x)[(\forall y)(Fyx \supset Fxy) \supset (\forall y)(Fxy \supset Fyx)]$

F. $(\forall x)(p \supset (\exists z)(Fxyz))$

1. $DQ = \{0, 1, 2, \dots\}$
2. $p := \top$
3. $y :=$ 6".
4. $extF = \{< x, y, z > | - (y < x \ . \ x < z\}$, i.e., the set of all ordered triples of numbers such that x is not between y and z.

G. $(\forall x)(Cx.(\exists y)(Px. - Lyx) \supset (\exists y)(Px.Lyx))$

H. $(\exists x)(Cx.(\forall y)(Cy \supset -Lyx))$