Philosophy 230

Wesleyan University
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Handout 3a

Monadic Schemata and Interpretation

I. Classification of monadic schemata

A. (*Monadic*) *open schemata*:

1. (*Monadic*) *atomic open schema*: e.g.,

$$Fx,\ Gy, \text{ etc.}$$

2. A (*monadic*) *complex open schema*: e.g.,

$$Fx.Gx \supset Hx$$

3. Note that ***only one*** free variable is allowed.

B. *Simple monadic schema*: e.g.,

$$(\exists x)(Fx \equiv Gx),$$

$$(\forall y)(Hy.(Gy \vee Fy))$$

C. *Pure monadic schema*: e.g.,

$$(\exists x)(Fx \equiv Gx) \supset (\forall y)(Hy.(Gy \vee Hy))$$

D. *Monadic schema*: e.g.,

$$(\forall x)(Fx \supset Gx \vee Hx) \supset Fx$$

II. The interpretation of monadic schemata: motivation.

A. In order to do anything with schemata, we need, as in the TF case, some notion of the interpretation of a schema.

B. Remember that in TF logic, we did truth tables for schemata.

1. Each row of a truth table is an interpretation.
2. And each row represents a possible situation, or state of the world.

C. So, the totality of rows of a truth table represents all the possible situations or states of the world in which a schema could be evaluated for truth or falsity.

D. And that is why we can be so confident that if we look at all the rows of a truth table, we know ***every possible way*** in which a schema could be $\top$ or $\bot$.

E. And this is what we are going to try to do for monadic schemata. We want an interpretation of a monadic schema to specify a possible state of the world that would determine the schema as $\top$ or $\bot$.

III. The interpretations of monadic schemata: summary of introductory discussion

A. What do we need to specify a possible state of the world? Three things:

1. what entities there are in the world,

2. what properties these entities have, and
3. which of these entities we are specifically talking about.

B. That is, an interpretation of monadic schemata must have:

1. A specification of what the universe of discourse or domain of quantification contains.
2. A specification of what the free variable(s) refer to.
3. A specification of what predicate letters refer to.

C. There are some technical requirements on the domain

1. Non-empty
2. Kept the same throughout a sentence or a group of sentences.

D. There are two kinds of interpretations: by replacement and by assignment.

IV. Interpretation by replacement

A. The universe of discourse is interpreted as the extension of a monadic predicate of English. So the specification of the domain consists of providing an English predicate with a non-empty extension.

B. Each monadic predicate letter which occurs in the schema is interpreted by a monadic predicate of English *which applies to objects in the domain.*

C. Each free variable which occurs in the schema, if any, is interpreted by some *name* of an object *which is in the universe of discourse.*

D. Once we have such an interpretation, we can translate a schema back into English.

E. And, once we have the English translation, we can determine whether the schema is $\top$ or $\perp$ by seeing whether the English sentence is $\top$ or $\perp$.

V. Interpretations by replacement

A. $(\forall x)(Fx \supset Gx \vee Hx) \supset Fx$

B. $(\forall x)(Fx \supset Gx).(\exists x)(Fx) \supset (\exists x)(Gx)$

VI. Interpretation by assignment

A. The domain of quantification is specified by a non-empty set of objects.

B. For each predicate letter, a specification of an extension, which must be a *subset* of the DQ.

C. An assignment of some element of the DQ to the free variable.

D. The three items that make up an interpretation by assignment is called a structure

E. Once we have such an interpretation, we can determine the truth value of the schema.

VII. Truth of a schema under an interpretation by assignment

We now define what it means for a monadic schema to be *true* under an interpretation by assignment. The definition here is based very closely on the definitions we have given of the truth conditions of sentences containing the quantifiers.

A. A simple open schema "Fx" is $\top$ if the object assigned to "x" is a member of the set assigned as extension to "F".

B. Whether a complex monadic open schema is $\top$ is determined by whether each of its simple parts is $\top$, and by the rules for using the TF connectives.

C. A simple monadic schema "$(\forall x)(\ldots\ x \ldots)$" is $\top$ if the monadic open schema contained in it is $\top$ for every assignment of an object in the UD to "x".

D. A simple monadic schema "$(\exists x)(\ldots\ x \ldots)$" is $\top$ if the monadic open schema contained in it is $\top$ for some assignment of an object in the UD to "x".

E. Whether a monadic schema is $\top$ is determined by whether each of the simple monadic schemata it contains is $\top$, whether each open schema it contains is $\top$, together with the rules for using the TF connectives.

VIII. Examples of truth-values under an interpretation by assignment

A. $(\forall x)(Fx \supset Gx \vee Hx) \supset Fx$

1. $UD = \{0, 1, ...\}$.
2. $extF = \{0, 2\}$
3. $extG = \{0\}$
4. $extH = \{2\}$
5. $x := 1$.

B. Find an interpretation of $(\forall x)(Fx \supset Gx).(\exists x)(Fx) \supset (\exists x)(Gx)$ that makes it $\top$ and one which makes it $\perp$.