

I. Restricted and unrestricted universes of discourse

A. Effects of differences in universes

Something in the garden is cold.

B. Consider:

1. $(\exists x)(x \text{ is in the garden. } x \text{ is cold})$
domain = the class of all people. And,
2. $(\exists x)(x \text{ is a person. } x \text{ is in the garden. } x \text{ is cold})$
domain = all objects.

C. Existential quantifications:

1. $(\exists x)(\dots x \dots)$,
with domain given by the extension of a predicate “ $F\textcircled{D}$ ”, and
2. $(\exists x)(Fx.(\dots x \dots))$
with domain unrestricted, i.e., all objects, have the same truth conditions.

D. Universal quantifications:

1. $(\forall x)(\dots x \dots)$,
with domain given by the extension of predicate “ $F\textcircled{U}$ ”, and
2. $(\forall x)(Fx \supset (\dots x \dots))$
with domain unrestricted, i.e., all objects have the same truth conditions.

Note that in the preceding two sections “ $(\dots x \dots)$ ” indicates any monadic schema with free variable “ x ”.

II. More quantification paraphrases:

A. Differences between “any” and “every” in negations:

1. Malone does not read **any** book recommended to him.
2. Malone does not read **every** book recommended to him.

B. Differences between “any” and “every” in antecedents of conditionals:

1. If **every** Senator attends, the party will be a success.
2. If **any** Senator attends, the party will be a success.

C. Cross-reference from antecedent to consequent:

1. If something I bought malfunctions, I’ll be annoyed.
2. If something I bought malfunctions, I’ll return it.

D. Logically complex antecedents:

1. All suggestions and comments should be addressed to the professor
2. All suggestions or comments should be addressed to the professor.
3. The indefinite articles: “a” and “an”:
An athlete will improve just in case she eats well.

III. Monadic Schematization with Predicate letters

A. To go from the paraphrases:

1. $(\forall x)((x \text{ is a man}) \supset (x \text{ is mortal}))$
2. $(\exists x)((x \text{ is German}).(x \text{ is a logician}))$

to a schematization of this argument, we introduce *predicate letters*:

- “ $M\textcircled{1}$ ” to replace “ $\textcircled{1}$ is a man”
- “ $O\textcircled{1}$ ” to replace “ $\textcircled{1}$ is mortal”,
- “ $G\textcircled{1}$ ” to replace “ $\textcircled{1}$ is German”, and
- “ $L\textcircled{1}$ ” to replace “ $\textcircled{1}$ a logician”.

B. We use the letters in the following way:

1. Instead of writing “ x is a man”, we write “ Mx ”
2. “ Ox ” instead of “ x is mortal”, etc.

C. The paraphrases then become monadic quantificational schemata:

1. $(\forall x)(Mx \supset Ox)$
2. $(\exists x)(Gx.Lx)$

where

- “ $M\textcircled{1}$ ” replaces “ $\textcircled{1}$ is a man”
- etc. . .

D. LET ME EMPHASIZE THAT HERE, AS IN THE CASE OF TRUTH FUNCTIONAL PARAPHRASE, YOU HAVE TO SAY WHAT YOUR PREDICATE LETTERS STAND FOR.

IV. Polyadic predicates

A. English statements containing polyadic predicates:

1. Tony likes Alex
2. Alex likes Alex

B. What these two have in common is:

$\textcircled{1}$ likes $\textcircled{2}$

C. *Open Sentences* are formed by putting free variables into the placeholders of a predicate:

x likes y

x likes x

D. *Predicate Letters*

1. If we introduce the predicate letter “ $L①②$ ” to schematize “① likes ②”, then the two open sentences above are schematized as

$$Lxy$$

$$Lxx$$

2. On the other hand, the open sentence

$$y \text{ likes } x$$

is schematized as

$$Lyx$$

V. Nested quantification

- A. What is the paraphrase of

Everyone loves someone?

- B. From this English statement we can infer:

1. Alex loves someone
2. Steve loves someone
3. Tony loves someone

- C. So, using monadic quantification, we paraphrase this sentence as:

$$(\forall x)(x \text{ loves someone})$$

- D. But “① loves someone” contains logical structure which hasn’t been made explicit, since it contains the word “someone”.

- E. Again using monadic quantification, we translate the open sentence

$$x \text{ loves someone}$$

as:

$$(\exists y)(x \text{ loves } y)$$

- F. So the paraphrase of the predicate “① loves someone” is:

$$(\exists y)(① \text{ loves } y)$$

- G. The full translation of our original sentence:

$$(\forall x)(\exists y)(x \text{ loves } y)$$

- H. It is **WRONG** to translate the original sentence as:

$$(\exists x)(\text{everyone loves } x)$$

WHY?

VI. Relative scopes and differences in truth conditions

- A. Everyone loves someone.
- B. Someone is loved by everyone.

VII. Logical Forms

A. Every statement has one of seven kinds of logical forms:

1. $(\forall x)(\dots x \dots)$
2. $(\exists x)(\dots x \dots)$
3. $\neg(\dots)$
4. $(\dots).(\dots)$
5. $(\dots) \vee (\dots)$
6. $(\dots) \supset (\dots)$
7. $(\dots) \equiv (\dots)$

B. If a statement has one of the first two of these forms, then we say that it has a *quantificational* form at the highest level.

C. In particular, we say that it is a universal quantification, or an existential quantification at the highest level.

D. If a statement has the next 5 forms, then we say that it has a *truth-functional* form at its highest level.

E. In particular, we say that the statement is a negation, or conjunction, or disjunction, or conditional, or biconditional, at the highest level.

VIII. Polyadic paraphrase:

If John likes someone, then everyone does.

IX. Definition of monadic schemata

A. (*Monadic*) *open schemata*:

1. (*Monadic*) *atomic open schema*: e.g.,
 Fx, Gy , etc.
2. A (*monadic*) *complex open schema*: e.g.,
 $Fx.Gx \supset Hx$
3. Note that **only one** free variable is allowed.

B. *Simple monadic schema*: e.g.,

$$(\exists x)(Fx \equiv Gx),$$
$$(\forall y)(Hy.(Gy \vee Fy))$$

C. *Pure monadic schema*: e.g.,

$$(\exists x)(Fx \equiv Gx) \supset (\forall y)(Hy.(Gy \vee Hy))$$

D. *Monadic schema*: e.g.,

$$(\forall x)(Fx \supset Gx \vee Hx) \supset Fx$$