Philosophy 230

Wesleyan University
Fall 2014

Handout 1a

Review of Basic Concepts and Truth-Functional Logic

I. Valid Patterns of Argument

Argument 1

A. If it rained, then the match was cancelled.
B. The match was not cancelled.
C. So, it didn't rain.

Argument 2

A. If I had gone to law school, then I would now lack civility.
B. I don't now lack civility.
C. Therefore, I hadn't gone to law school.

Argument 3

A. If I had gone to law school, then the match was cancelled.
B. The match was not cancelled.
C. So, I had not gone to law school.

Underlying Form 1

A. If p then q
B. It is not the case that q
C. Therefore, it is not the case that p

Argument 4

A. All mammals are warm-blooded.
B. Roscoe is a mammal.
C. So, Roscoe is warm-blooded.

Argument 5

A. All Italian operas end tragically.
B. *Rigoletto* is an Italian opera.
C. Therefore, *Rigoletto* ends tragically.

Underlying Form 2

A. All Fs are Gs

B. x is an F

C. Therefore, x is a G

II. Paraphrases

A. Logical Constants

Truth-functional connectives

"." for "and",
"$\vee$" for "or",
"$-$" for "it is not the case that",
"$\supset$" for "if ... then $\cdots$",
"$\equiv$" for "... just in case $\cdots$"

Quantifiers

"$\forall$" for "all"
"$\exists$" for "some".

B. Two more arguments of form 1:

Argument 6

1. North will not testify unless he is granted immunity.
2. North is not granted immunity.
3. So, North will not testify.

Argument 7

1. North will testify only if he is granted immunity.
2. North is not granted immunity.
3. So, North will not testify.

C. Two more arguments of form 2:

Argument 8

1. Each soprano sang a C.
2. Angela is a soprano.
3. So, Angela sang a C.

Argument 9

1. Contraltos envy counter-tenors.
2. Lorraine is a contralto.
3. Therefore, Lorraine envies counter-tenors.

D. Paraphrase of the first premise of arguments 6 and 7:

(North will testify) $\supset$ (North is granted immunity)

E. Paraphrase of the first premise of arguments 8 and 9:

1. $(\forall x)((x \text{ is a soprano}) \supset (x \text{ sang a C}))$
2. $(\forall x)((x \text{ is a contralto}) \supset (x \text{ envies counter-tenors}))$

III. Standard Truth-Functional Paraphrases

p only if q	$p \supset q$
p provided that q	$q \supset p$
p unless q	$-p \supset q$ or $p \vee q$
p if and only if q	$p \equiv q$
p just in case q	$p \equiv q$
p even though q	$p.q$

IV. Schematization

A. Three statements with the same *type* of truth conditions:

1. (Clinton was impeached).(Clinton was removed from office)
2. (Bush declared his presidential candidacy).(Bush raised no campaign money)
3. (Clinton was impeached).(Bush raised no campaign money)

B. Schematization of these statements:

$$p.q$$

C. Another three statements with the same *type* of truth conditions:

1. $(\forall x)((x \text{ is a prime numbers}) \supset (x \text{ is odd}))$
2. $(\forall x)((x \text{ is a mammal}) \supset (x \text{ is a land-dweller}))$
3. $(\forall x)((x \text{ is a perfect number}) \supset (x \text{ is odd}))$

D. Schematization of these statements:

$$(\forall x)(Fx \supset Gx)$$

E. Schematization of arguments 1-3, 6 and 7:

1. $p \supset q$
2. $-q$
3. Therefore, $-p$

F. Schematization of arguments 4, 5, and 8-9:

1. $(\forall x)(Fx \supset Gx)$
2. Fx
3. Therefore, Gx

V. Interpretations of Schemata in Argument-Form 1

A. Interpretations of schemata are represented by truth-tables:

p	q	$p \supset q$	$-q$	$-p$
$\top$	$\top$	$\top$	$\bot$	$\bot$
$\top$	$\bot$	$\bot$	$\top$	$\bot$
$\bot$	$\top$	$\top$	$\bot$	$\top$
$\bot$	$\bot$	$\top$	$\top$	$\top$

B. Each row of the truth-table is an interpretation; each interpretation is an assignment of truth-value, "$\top$" for truth, "$\bot$" for falsity, to each of the sentence-letters in the schema.

VI. Logical Properties of Schemata

A. **Implication**, or **Semantic Logical Consequence**: a set of schemata $A_0, \ldots, A_n$ *implies* a schema A, (or, A is a *logical consequence* of $A_0, \ldots, A_n$) if and only if, whenever all of $A_0, \ldots, A_n$ come out true under any interpretation I (written $I \vDash A_0, \ldots, A_n$), so does A (i.e., $I \vDash A$).

B. **Validity**: a schema A is *valid* iff (our abbreviation, from now on, for "if and only if") for every interpretation I, $I \vDash A$.

C. A slogan: implication is the validity of the conditional.

D. **Equivalence** applies to two schemata: A and B are **equivalent** just in case they have the same truth values under every interpretation; i.e., for any interpretation I, $I \vDash A$ iff $I \vDash B$.

E. A slogan: equivalence is the validity of the biconditional.

F. **Satisfiability**: a schema A is *satisfiable* iff there is an interpretation I such that $I \vDash A$.

G. **Invalidity**: a schema is *invalid* iff it is *not* valid. That is, A is invalid iff there's *at least one* interpretation I in which A is false (written $I \nvDash A$.)

H. **Unsatisfiability** is the negation of satisfiability, just as invalidity is the negation of validity. A schema A is *unsatisfiable* iff for *every* interpretation I, $I \nvDash A$.

I. An argument in ordinary language is valid just in case the schematization of its conclusion is implied by the schematizations of all its premises.

VII. First Two Questions from Quiz

A. I.B.7 (c) (p. 257-8) For the following argument, determine whether the premise truth-functionally imply the conclusion by schematizing the statements and providing a truth-table:

The police will act courageously and the pirates will be routed, if Fredric leads the attack; but if Fredric doesn't lead the attack then the police won't act courageously and the pirates won't be routed.

THEREFORE, The pirates will be routed if and only if Fredric leads the attack and the police act courageously.

B. I.C.1 (p. 260) Determine whether each of the following is true or false. If true, explain why it is; if false, provide a counter-example, i.e., a pair of truth-functional schemata that shows the generalization is false.

(a) Any schema implied by a satisfiable schema is satisfiable.

(b) Any schema implying a satisfiable schema is satisfiable.