Philosophy 230

Wesleyan University
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Handout 11a

Second Part of Soundnes Proof

I. Two ways of eliminating EII premises in the system with EI rules

A. EI premise eliminated by EIE

$$\vdots$$

$[i_1, \ldots, i_n]$	(l)	$(\exists x)\Phi(x)$	$\cdots$
		$\vdots$	
$[i_1, \ldots, i_n, k]$	(k)	$\Phi(y)$	$(l)yEII$
		$\vdots$	
$[j_1, \ldots, j_p, k]$	(o)	A	$\cdots$
		$\vdots$	
$[j_1, \ldots, j_p]$	(m)	A	$[k](o)EIE$

Note that in order for EIE to be applicable to (o), y is not free in A or in $S_{j_1}, \ldots, S_{j_p}$.

B. EI premise eliminated by D

$$\begin{array}{llll}
 & & \vdots & \\
[i_1,\ \ldots,\ i_n] & (l) & (\exists x)\Phi(x) & \cdots \\
 & & \vdots & \\
[i_1,\ \ldots,\ i_n, k] & (k) & \Phi(y) & (l)yEII \\
 & & \vdots & \\
[j_1,\ \ldots,\ j_p, k] & (o) & A & \cdots \\
 & & \vdots & \\
[j_1,\ \ldots,\ j_p] & (m) & \Phi(y) \supset A & [k](o)D
\end{array}$$

where again y does not occur free in $S_{j_1}, \ldots, S_{j_p}$.

II. Eliminating EII

In either of the two cases above, we do the same thing, namely, add a conditional premise, and deduce the schema that originally results from EII by TF instead:

$$\begin{array}{llll}
 & & \vdots & \\
[i_1,\ \ldots,\ i_n] & (l) & (\exists x)\Phi(x) & \cdots \\
 & & \vdots & \\
[k] & (k) & (\exists x)\Phi(x) \supset \Phi(y) & P \\
[i_1,\ \ldots,\ i_n, k] & (k\frac{1}{2}) & \Phi(y) & (l)(k)TF \\
 & & \vdots &
\end{array}$$

III. Replacement of the elimination of the EI premise at line (m)

This replacement depends on which of the two forms the original deduction has.

A. Line (m) obtained by D:

$$\vdots$$

$[i_1, \ldots, i_n]$	(l)	$(\exists x)\Phi(x)$	$\cdots$
		$\vdots$	
$[k]$	(k)	$(\exists x)\Phi(x) \supset \Phi(y)$	P
$[i_1, \ldots, i_n, k]$	$(k\frac{1}{2})$	$\Phi(y)$	$(l)(k)TF$
		$\vdots$	
$[j_1, \ldots, j_p, k]$	(o)	A	$\cdots$
		$\vdots$	
$[j_1, \ldots, j_p]$	(m)	$[(\exists x)\Phi(x) \supset \Phi(y)] \supset A$	$[k](o)D$
$[j_1, \ldots, j_p]$	$(m\frac{1}{2})$	$\Phi(y) \supset A$	$(m)TF$

$$\vdots$$

B. Line (m) obtained by EIE:

$\vdots$

$[i_1,\ \ldots,\ i_n]$	(l)	$(\exists x)\Phi(x)$	$\cdots$

$\vdots$

$[k]$	(k)	$(\exists x)\Phi(x) \supset \Phi(y)$	P
$[i_1,\ \ldots,\ i_n, k]$	$(k\frac{1}{2})$	$\Phi(y)$	$(l)(k)TF$

$\vdots$

$[j_1,\ \ldots,\ j_p, k]$	(o)	A	$\cdots$

$\vdots$

$[j_1,\ \ldots,\ j_p]$	(m)	$[(\exists x)\Phi(x) \supset \Phi(y)] \supset A$	$[k](o)D$
$[m+1]$	$(m+1)$	$-A$	P
$[j_1,\ \ldots,\ j_p, m+1]$	$(m+2)$	$(\exists x)\Phi(x)$	$(m)(m+1)TF$
$[j_1,\ \ldots,\ j_p, m+1]$	$(m+3)$	$-\Phi(y)$	$(m)(m+1)TF$
$[j_1,\ \ldots,\ j_p, m+1]$	$(m+4)$	$(\forall x) - \Phi(x)$	$(m+3)UG$
$[j_1,\ \ldots,\ j_p, m+1]$	$(m+5)$	$-(\exists x)\Phi(x)$	$(m+4)CQ$
$[j_1,\ \ldots,\ j_p, m+1]$	$(m+6)$	A	$(m+2)(m+5)TF$
$[j_1,\ \ldots,\ j_p]$	$(m+7)$	$-A \supset A$	$[m+1](m+6)D$
$[j_1,\ \ldots,\ j_p]$	$(m+8)$	A	$(m+7)TF$

$\vdots$

IV. Example of elimination of EII and EIE

A. The original deduction is:

[1]	(1)	$(\exists x)Fx$	P
[1, 2]	(2)	Fa	$(1)xEII$
[1, 2]	(3)	$Fa \vee Ga$	$(2)TF$
[1, 2]	(4)	$(\exists x)(Fx \vee Gx)$	$(3)EG$
[1]	(5)	$(\exists x)(Fx \vee Gx)$	$[2](4)EIE$

B. First we eliminate the application of EII on line 2, thus, in this example $k = 2$:

[1]	(1)	$(\exists x)Fx$	P
[2]	(2)	$(\exists x)Fx \supset Fa$	P
[1, 2]	$(2\frac{1}{2})$	Fa	$(1)(2)TF$

C. Now we continue the deduction, just as before, up to the point where EIE was originally used, i.e., (5), but now continue by discharging (2) (note that this means that in this example $m = 5$):

[1, 2]	(3)	$Fa \vee Ga$	$(2)TF$
[1, 2]	(4)	$(\exists x)(Fx \vee Gx)$	$(3)EG$
[1]	(5)	$[(\exists x)Fx \supset Fa] \supset (\exists x)(Fx \vee Gx)$	$[2](4)D$

D. And we finish with a *reductio ad absurdum* argument:

[6]	(6)	$-(\exists x)(Fx \vee Gx)$	P
[1, 6]	(7)	$(\exists x)Fx$	$(5)(6)TF$
[1, 6]	(8)	$-Fa$	$(5)(6)TF$
[1, 6]	(9)	$(\forall x) - Fx$	$(8)UG$
[1, 6]	(10)	$-(\exists x)Fx$	$(9)CQ$
[1, 6]	(11)	$(\exists x)(Fx \vee Gx)$	$(7)(10)TF$
[1]	(12)	$-(\exists x)(Fx \vee Gx) \supset (\exists x)(Fx \vee Gx)$	$[6](11)D$
[1]	(13)	$(\exists x)(Fx \vee Gx)$	$(12)TF$