I. Deontic Logic, the concept of Moral Obligation, and Fundamental Deontic Principles
   A. The logic of obligation and permission is called deontic logic. Just as in epistemic logic, we will introduce some new symbols: ‘O’ for (moral) obligation, and ‘P’ for (moral) permission.
   B. The following two are NOT principles of deontic logic:
      1. O\(p \Rightarrow p\)
      2. P\(p \Rightarrow p\)
   C. Obligation and permission are interdefinable.
      1. What does it mean for an act to be permitted? It means that you don’t have to stop yourself from doing it. That is, you’re not obligated not to do it. So permission can be defined in terms of obligation.
         \(Pp \equiv \neg O(\neg p)\)
      2. The reverse also holds. If an action is morally required, then you’re not allowed not to do it. That is, you’re not permitted not to do it.
         \(Op \equiv \neg P(\neg p)\)
   D. Moreover, there is another connection between obligation and permission that seems inescapable. One cannot be obligated to do something, and yet, at the same time, not be permitted to do it. A system of morality that obligates and forbids the same action would not be consistent. So we have a Principle of Deontic Consistency, if one is obligated to do something then one is permitted to do it.
         \(Op \Rightarrow Pp\)
   E. Given the definition of permission in terms of obligation, we can also formulate this principle as
      \(Op \Rightarrow O(\neg p)\)
II. More Controversial Deontic Principles
   A. The Agglomeration Principle: if you ought to do something, \(p\), and, you also ought to do another thing, \(q\), then you ought to do them both, i.e., do \(p.q\).
      \((Op.Oq) \Rightarrow O(p.q)\)
   B. The Ought Implies Can Principle: if it is not possible for one to do something, then it doesn’t seem that one should be required to do it. The idea of excuse or exculpating circumstances depends at least in part on this. If, for example, I promised to lend you my car tomorrow, but someone steals it tonight, then my not lending you my car tomorrow is excusable, I wouldn’t have failed to do what I should, because given the theft I couldn’t have lent it to you. We’ll use the new symbol ‘O(…’ for ‘it is possible that …’, and formulate this principle as
      \(Op \Rightarrow \diamond p\)
   C. Principle DK (Deontic K). This principle concerns necessary consequences or presuppositions of
our actions. For example, since it’s a law of physics that no body can occupy two places at the same time, if I go to California to visit my parents, then necessarily I miss classes in Connecticut. Does it make sense to think it could be morally required of me to visit my parents, but not obligatory to miss my classes at the same time? Suppose I’m not required to miss those classes; then, obviously, I may go to those classes. But by going to those classes I necessarily am not visiting my parents. So surely it’s permitted for me not to visit my parents. But how could that be, if I’m morally required to visit them? The general principle that corresponds to this reasoning is

\[
\neg \Diamond(p \land \neg q) \supset (O p \supset O q)
\]

III. Alethic modalities

A. The notion of possibility that we have just introduced is clearly also a mode of truth: it contrasts with what is actually true and what is necessarily true. These concepts are called the alethic modalities. We introduce another new symbol, ‘\(\Box\)’ for ‘it is necessary that …’. Necessity stands to possibility as obligation stands to permission. That is, if \(p\) is necessary, then it’s not the case that \(p\) might not be the case; similarly, if \(p\) is possible, then it’s not necessary that \(p\) is not the case:

1. Def\(\Box\): \(\Box p \equiv \neg \Diamond(\neg p)\) or \(\neg \Box p \equiv \Diamond(\neg p)\)
2. Def\(\Diamond\): \(\Diamond p \equiv \neg \Box(\neg p)\) or \(\neg \Diamond p \equiv \Box(\neg p)\)

B. For our purposes we interpret necessity and possibility logically. That is, a statement or schema is necessary just in case it is valid, and possible just in case it is satisfiable. This leads to two forms of reasoning about necessity and possibility.

1. A logical truth is a statement with a valid schema. Now, as I hope you’ll remember, a valid schema is true under every interpretation. Since an interpretation represents one possible way the statement can be determined as true or false, a schema is valid just in case there is no possible way for it to be false. That is, it is necessarily true. So from an argument showing a statement to be logically true we can infer that that statement is necessarily true. The rule of inference is called Necessitation.

2. The other rule is the analogue for alethic modality of the rule K of epistemic logic. Suppose a schema \(X\) implies a schema \(Y\). Then, as we now know, \(X \supset Y\) is valid, and so necessarily true, i.e., \(\Box(X \supset Y)\) is true. Now suppose, in addition, that \(X\) is valid, i.e., \(\Box X\) is true. From our work in truth-functional logic we know that \(Y\) must also be valid, i.e., from these suppositions it follows that \(\Box Y\) is true. Thus we have the principle:

\[
\Box(X \supset Y) \supset (\Box X \supset \Box Y)
\]

C. Using these two principles together, we can see that if \(X\) truth-functionally implies \(Y\), then \(\Box X\) implies \(\Box Y\). Here’s the argument. If \(X\) implies \(Y\), then \(X \supset Y\) is valid, so \(\Box(X \supset Y)\) is true. Principle K and MP allows us to infer \(\Box X \supset \Box Y\). On the assumption that \(\Box X\) is true, it follows again by MP that \(\Box Y\) is true. We’ll call this Transfer of Necessity.

D. Now, let’s use these forms of inference to deduce another formulation of principle DK.

1. \(\neg \Diamond(p \land \neg q)\) is equivalent to \(\Box(\neg(p \land \neg q))\).
2. But \(\neg(p \land \neg q)\) is truth-functionally equivalent to \(p \supset q\).
3. Hence \(\Box(p \supset q)\) is equivalent to \(\neg \Diamond(p \land \neg q)\). So DK can equivalently be stated as

\[
\Box(p \supset q) \supset (O p \supset O q)
\]

IV. Two arguments against the existence of moral dilemmas.
A. Premises (1), (2), and (3) represent the claim that moral dilemmas exist.

B. The deduction of the first argument:

1. $O_p$  
2. $O_q$  
3. $\neg \diamond (p.q)$  

4. $\neg \diamond (p.q) \Rightarrow \Box (\neg (p.q))$  
5. $\Box (\neg (q.p))$  
6. $\Box (q \supset p)$  

7. $O_q \supset O(\neg p)$  
8. $O(\neg p)$  
9. $O(\neg p) \Rightarrow O(\neg p)$  

10. $O(\neg p)$  
11. $\neg O(\neg p)$  
12. $O(q \supset p) \Rightarrow (O_q \supset O(\neg p))$  

V. The Good Samaritan Paradox.

A. If you come across someone who has just been robbed, then it seems that the right thing for you to do is help her to the extent that you can. (And if you do help her, then you’re a good samaritan; hence the name of the paradox.) Now it seems that this moral obligation can be expressed as follows:

It is obligatory that you help Hillary who has been robbed.

i.e.,

$O(you\ help\ Hillary\ who\ has\ been\ robbed)$

B. But your helping Hillary who has been robbed consists in two things:

1. You help Hillary, and
2. Hillary was robbed.

C. But a conjunction implies each conjunct, so the claim that if you help Hillary who has been robbed then Hillary has been robbed, i.e.,

$(You\ help\ Hillary\ who\ has\ been\ robbed) \supset (Hillary\ has\ been\ robbed)$

is logically true, and so is necessary:

$\Box ((You\ help\ Hillary\ who\ has\ been\ robbed) \supset (Hillary\ has\ been\ robbed))$

D. Principle DK has as an instance:

$\Box ((You\ help\ Hillary\ who\ has\ been\ robbed) \supset (Hillary\ has\ been\ robbed)) \Rightarrow (O(you\ help$
E. So by applying MP twice, one can conclude

\[ O(\text{Hillary has been robbed}) \]

i.e.

It is obligatory that Hillary be robbed.

This is a highly counter-intuitive result.