Russell’s Last (and Best) Multiple-Relation Theory of Judgement

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Russell’s version of the multiple-relation theory from the Theory of Knowledge manuscript is presented and defended against some objections. A new problem, related to defining truth via correspondence, is reconstructed from Russell’s remarks and what we know of Wittgenstein’s objection to Russell’s theory. In the end, understanding this objection in terms of correspondence helps to link Russell’s multiple-relation theory to his later views on propositions.

1. Introduction

Russell first presented a version of the multiple-relation theory of judgement in the 1906 paper ‘The Nature of Truth’ (Russell 1906). It figured in the paper as a viable alternative to what could be called a dual-relation theory. According to the dual-relation theory, a propositional attitude like judgement involves a basic relation between a judging subject and complex thing called a proposition. Judgement is thus a two-place or dual relation. A central feature of the dual-relation theory is that the entities comprising the subject-matter of the judgement are constituents of the proposition, in the simplest possible sense of ‘constituent’. That is, the proposition has as parts the entities that the judgement is about. The same holds for a propositional attitude like judgement or belief. Russell conceives of propositional attitudes as complex things or complexes that have as parts not only the judging subject, but also the proposition judged. On the assumption that if $a$ is a part of $b$ and $b$ is a part of $c$ then $a$ is a part of $c$, we get:

\[(\text{PART})\text{ In a propositional attitude, the entities that are the subject-matter of this attitude are also parts of the propositional attitude}\]

1 Unless otherwise indicated, all references are to the page/lines of Russell 1983.
2 Russell 1906, p. 49: ‘As between the above two views of truth, I do not at present see how to decide.’
(PART) is such a crucial feature of Russell's conception of judgement that he maintains it even after giving up the dual-relation theory in favour of the multiple-relation theory. This shift, announced in the 1910 paper 'On the Nature of Truth and Falsehood' and *Principia Mathematica*, dispenses with propositions as complex things. (PART) is maintained, however, by presenting a propositional attitude like judgement as a many-place or multiple-relation between a judging subject and the entities that are the subject-matter of the judgement. The change is easier to grasp with an example. According to the dual-relation theory, when Othello judges that Desdemona loves Cassio, this involves

$$J(o, p)$$

where \( p \) is a complex thing, Desdemona's loving of Cassio.\(^3\) Alternatively, the multiple-relation theory requires something of the form

$$J(o, d, L, c)$$

where \( J \) is now a four-place relation between Othello, Desdemona, the loving relation, and Cassio. After the shift, nothing like \( p \) is required to give an account of propositional attitudes.

Besides (PART), there is another core commitment to Russell's two accounts of propositional attitudes. In both stages he defends a principle of bivalence:

\((T/F)\) Each judgement has exactly one of the following two properties: truth, falsity

On the dual-relation theory, \((T/F)\) took a particularly aggressive form, for Russell insisted that truth and falsity are simple properties of the complex things that he calls propositions. This has led some to call this approach to propositions an 'identity theory' (Baldwin 1991). According to the identity theory the object of a judgement is identical with a part of the judgement. Exactly why Russell would adopt an identity theory remains a debatable issue. It surely has something to do with Moore's early paper 'The Nature of Judgment' (1899) and its argument that any kind of correspondence theory of truth is untenable. However, the important point for this paper is that Russell remained committed to \((T/F)\) long after he abandoned the identity theory.\(^4\) At some point he must have realized that a correspondence theory of a certain special

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\(^3\) Along with Russell, I pretend that this judgement actually occurred and that Othello, Desdemona, and Cassio are real, historical figures.

sort would elude Moore’s objections. But he seems to have thought that
(T/F) remained essential. As we will see, the multiple-relation theory
requires a correspondence theory in order to specify which judgements
are true and which judgements are false. In line with (T/F), Russell does
not present this as an additional topic to his theory of judgement. Truth
and judgement go together for Russell, as the titles of his papers on
judgement readily attest.

Russell offers one clear motivation for rejecting the dual-relation
account in favour of the multiple-relation theory. He notes that the
dual-relation theory requires complex things that have the property of
being false in addition to the complex things that have the property of
being true. So, in addition to the complex thing invoked in the correct
judgement that Othello is jealous, there must also be what we called p
above: Desdemona’s loving of Cassio. This entity is united by the loving
relation, and it relates Desdemona to Cassio. But it is not true that
Desdemona loves Cassio, despite there being such a complex. Truth
would require that this complex thing have the simple property truth
attached to it, whereas in fact it has the property false. As these are sim-
ple properties, there is no more basic explanation of what the truth or
falsity of propositions depends on. Russell says he rejects the dual-rela-
tion theory because the idea of complexes that are false, or objective
falsehoods, ‘is in itself almost incredible’ (Russell 1910, p. 152). This
makes it sound like Russell is concerned with issues of ontological
economy. But we could press the point further by noting that it is quite
hard to posit an entity that is Desdemona standing in the loving rela-
tion to Cassio, but to also insist that Desdemona does not love Cassio.
Of course, this is just the identity theory that Russell had long
defended, so its strangeness was clearly not a decisive objection to it.
But if an alternative presented itself, it is understandable why Russell
would have explored it.

In addition to Russell’s explicit motivations for adopting the multi-
ple-relation theory, recent scholarship on Russell’s work on the founda-
tions of logic has drawn attention to Russell’s adoption of the theory of
types right around the time that he rejects propositions as complex
entities. A crucial piece of text linking the two changes comes in the
third section of ‘On the Nature of Truth’. After rehearsing the argument
that entails that ‘if beliefs always have objects, it follows that there are
objective non-facts’, Russell continues,

This argument would be conclusive, I think, if it were certain that a belief can
be validly regarded as a single state of mind. There are, however, difficulties
in so regarding a belief. The chief of these difficulties is derived from para-
doxes analogous to that of the liar, e.g., from the man who believes that all of his beliefs are mistaken, and whose beliefs are certainly all mistaken ... We can escape this paradox if a belief cannot be validly treated as a single thing. (Russell 1906, p. 46)

This shows that Russell may have pursued the multiple-relation theory in order to resolve his logical paradoxes.

A more direct argument against propositions as complex entities has recently been offered by Graham Stevens in his impressive *The Russellian Origins of Analytic Philosophy*. Stevens explains how the substitution theory which Russell initially tried to use to resolve the paradoxes generated its own paradox for propositions precisely because it treated propositions as complex entities (Stevens 2005, pp. 58–61). One way to block this paradox was to divide propositions into orders. Still,

How is one to justify the claim that entities differ in order? Substitution provides a neat explanation for types because the type of a matrix is simply determined by the number of its arguments; i.e., the number of entities contained in it. There is only one kind of entity as a result. The division of propositions into orders has no such justification. The ramified substitutional theory is formally adequate, but philosophically untenable. Russell’s problem is precisely this: the substitutional theory can only work if one maintains an ontology of propositions, but, as the paradoxes of propositions show, the admission of propositions requires the ramification of the substitutional theory. But the ramification of the substitutional theory conflicts with the very philosophical foundations that recommend the theory in the first place. Something had to give. It turned out to be the ontology of propositions on which the substitutional theory depends. (Stevens 2005, p. 63)

The steps in Stevens’s argument are not entirely clear, but we can isolate two assumptions that would entail his conclusion.

First, there is the claim that

(CAT) All entities fall into one metaphysical category

(CAT)’s roots run quite deep, back at least to *Principles of Mathematics* and what is called the doctrine of the unrestricted variable. There Russell argued that everything that is, or every term, must be the logical subject of some proposition. That is, there is an analysis of at least one proposition in which this term occurs according to which the proposition is about that term. The argument also applies to complex terms, which at this point in Russell’s metaphysics includes propositions as well as ‘aggregates’ which are distinguished from propositions (Russell 1903, Sect. 135). So, for example, the very same proposition that Othello judges is the subject of the following proposition: Desdemona’s love for
Cassio is much discussed. In fact, attributions of truth or falsity require this. In making the claim that Dedemona's love for Cassio is false, I must make this proposition the logical subject (Russell 1903, Sect. 52). Russell does not give any more enlightening account of what makes it the case that all terms fall into one metaphysical category, that is, why it is that every term is the logical subject of some proposition. But as we will see, he was eventually willing to give up (CAT) in favour of a two-category metaphysics which places things and facts in different categories. After this shift, Russell will deny that a fact is the subject of any proposition.

Still, a second, stronger, assumption is needed to get Stevens's conclusion:

(SUB) Replacing the logical subject of one proposition with the logical subject of another proposition always yields a proposition

In the theory of types propositional variables are divided into orders and propositional function variables are divided into orders and types. If we assume a dual-relation theory, then propositional variables range over genuine entities. According to (CAT), these entities must be the logical subject of some proposition. But then, by (SUB), any proposition can be substituted for any other proposition to produce a new proposition. This is precisely what the orders of the propositional variables rule out. For example, if \( F(P) \) and \( G(Q) \) are propositions whose logical subjects are propositions \( P \) and \( Q \), and \( P \) and \( Q \) are of different orders, then \( F(Q) \) and \( G(P) \) are not propositions. That is, the ramified theory of types requires that (SUB) fails. Combining (CAT), (SUB), and the ramified theory of types entails a contradiction. So, propositions are not entities.

Unfortunately, the status of (SUB) remains controversial. Some commentators, such as Linsky, might deny (SUB) and use this to block the above argument. As a result Linsky can maintain that even after Russell adopted the theory of types, propositional variables ranged over some metaphysically derivative entity (Linsky 1999, Sect. 2.2). In what follows I want to remain neutral on this complex debate. I will rely only on (CAT) and argue that even this weak principle was given up as a result of problems with the multiple-relation theory.

Although Russell's arguments about the unrestricted variable are central to his philosophy in this period, it is important not to overstate what they show. They do not show, and Russell explicitly repudiates, the stronger claim that every term is the logical subject of every proposition in which it occurs. The failure of this stronger claim grounds
Russell's metaphysical distinctions in *Principles* between things and concepts. A concept like humanity is not the logical subject of the proposition expressed by 'Socrates is human' on any of its analyses. It is the subject of other propositions, like that expressed by 'Humanity is possessed by Socrates' (Russell 1903, Sect. 48). In order to avoid confusing this distinction between things and concepts and that found in the later theory of types, I will say that things and concepts are of different kinds. (CAT) is consistent with different kinds of entities, but not with entities of more than one category. This is just to say that we can make distinctions within a single metaphysical category based on the combinatorial properties of entities.

My primary goal in this paper is to present Russell's most fully considered version of the multiple-relation theory and to argue that it conforms to the restrictions encapsulated in (PART), (T/F), and (CAT). Once this is clear, I will also argue that all the standard objections to Russell's multiple-relation theory fail, but that an objection that is hardly ever discussed is decisive. Much of the focus on the failings of the multiple-relation theory concerns Wittgenstein's objections and how they caused Russell to abandon the theory. In section six, I will review what is known of these objections and how they accord with the objection I will present to the multiple-relation theory.

To organize my discussion, I present Russell as trying to use the multiple-relation theory to solve a problem about the nature of propositional attitudes that has two distinguishable parts. This problem is to say, for each propositional attitude, what its essential features are. To do this, Russell must first say what each propositional attitude's composition is: what are its parts and how are these parts related. I call this the proposition problem. Second, Russell must say what relations a given propositional attitude must stand in if it is to be that kind of propositional attitude. For some propositional attitudes, like judgement or belief, these relations include correspondence relations which fix the truth-value of the propositional attitude. I call this aspect of the propositional attitude problem the correspondence problem. We will see that these aspects of the propositional attitude problem are not sufficiently distinguished in some objections to the multiple-relation theory, but that distinguishing them allows us to see that the correspondence problem is the most difficult problem to solve.

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5 Here I am obviously using 'kind' somewhat differently than Stevens is in the above quotation.

6 Although Russell begins by focusing on judgement, by 1913 he seems to prefer to talk about belief. I will treat these as interchangeable in this paper.
2. The theory of complexes

In this section and the next I will present Russell’s most mature version of the multiple-relation theory. This version was given in the Theory of Knowledge manuscript of 1913, which Russell abandoned in response to Wittgenstein’s objections. My presentation is divided into two parts. In this section I summarize what I will call Russell’s theory of complexes. This is a general metaphysical theory which outlines what sorts of complex things Russell envisages, and how they can be distinguished from one another. In line with the general metaphysical principle (CAT) all of these complexes are entities that fall into one metaphysical category. In Theory of Knowledge Russell’s metaphysical claims are constrained by the things that he thinks we can know about via acquaintance, the basic cognitive relation of his account. Still, almost all of the points I present in this section are motivated independently of the theory of judgement.\(^7\) As in Principles several kinds of entities are discussed, although their connections end up being richer than anything Russell had countenanced in 1903. Still, the model for complexes remains the same as what we saw above, that is, a thing having a property or things standing in relations.\(^8\) The most important difference is that only the true complexes of Principles survive as complexes in Theory of Knowledge. False complexes, and the conception of truth and falsity as simple properties of complex things, are now rejected.

To start, Russell introduces a distinction between permutative and non-permutative complexes (p. 144).\(^9\) Non-permutative complexes are those whose constituents can only form a complex in one way, while the constituents of permutative complexes can form more than one complex. Here are three of Russell’s favourite examples: ‘A is similar to B’, ‘A is before B’, and ‘A is a part of α’. The first statement involves a complex with constituents A, B, and the relation of similarity. For Russell, these constituents can only form one complex, and this leads him to the somewhat paradoxical conclusion that A and B occupy the same position in such a complex. This does not mean, I hope, that A and B must be somehow co-located. It is rather only that in so far as we want to distinguish complexes, there is no distinguishing A’s being similar to B from B’s being similar to A. When shifting constituents around in this

\(^7\)See esp. pp. 79–81 and pp. 122–8.
\(^8\)The aggregates from Principles drop out of Russell’s metaphysics fairly quickly.
\(^9\)Here Russell introduces these terms to apply to belief complexes specifically. But his talk of ‘permutation groups’ at p. 123/15 suggests the terminology can be applied more generally.
way fails to produce a new complex, Russell calls the pair of constituents *symmetrical*. In all other cases, the pair is *unsymmetrical*.

When we compare *A*’s being before *B* with *B*’s being before *A*, we note that we have two different complexes. So *A* and *B* are here unsymmetrical. But unlike some pairs of constituents, substituting *A* and *B* gives us a new complex, rather than something that fails to form a complex. As Russell puts it, *A* and *B* are here *homogeneous*. Crucially, when we try to substitute the relation of similarity with *A*, we fail to get a complex. That is, this pair of constituents are *heterogeneous*. More generally, any term like *A* is heterogeneous with a property, a dual relation, a three-place relation, etc. And, it seems, if we were to have a complex with both a property and a dual relation along with other constituents, the property and the dual relation would also be heterogeneous.

While these sorts of distinctions seem broadly logical and unproblematic, Russell goes on to argue that when *A* is a part of *α*, *A* and *α* are heterogeneous. The sense in which there is not or cannot be a complex where *α* is a part of *A* is not really explained by Russell.\(^\text{10}\) It seems to be a kind of brute metaphysical fact that he feels entitled to appeal to. It might seem like the distinction here requires ultimate metaphysical simples. But Russell’s clarification of ‘the form of dual complexes consisting of a simple and a complex (i.e. a relatively simple and a relatively complex)’ (p. 135/6–8) shows that only relative simplicity is involved.

These distinctions allow us to flesh out in more detail the prior division between permutative and non-permutative complexes. For a complex to be non-permutative, for any pair of constituents, the pair must be either (1) symmetrical or (2) heterogeneous. Otherwise, the complex is permutative.

At the heart of this entire theory is a distinction between descriptions that pick out a logically possible complex and those that fail to do this. Unfortunately, rather late in the manuscript, Russell says ‘We call a complex “logically possible” when there is a corresponding proposition’ (p. 122/31–3). This cannot be the whole story because Russell no longer believes in propositions and, as we will see shortly, is in fact using his account of complexes in order to explain what understanding, belief, etc. complexes really are. And earlier, fortunately, we find Russell admitting just this point:

When we were discussing relations, we said that, with a given relation and given terms, two complexes are ‘logically possible’. But the notion of what is

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\(^{10}\) See p. 123/5–6 and p. 135/5–10. In the former passage Russell talks of the ‘constituent’ of a complex, while in the latter he uses ‘part’. It does not seem that Russell distinguishes these two relations between a thing and a complex.
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‘logically possible’ is not an ultimate one, and must be reduced to something that is actual before our analysis can be complete. Now although we do not yet know what a proposition is, it is obvious that what we had in mind, when we said that a complex was ‘logically possible’, may be expressed by saying that there is a proposition having the same verbal form. This is still not ultimate, because of our doubt as to how propositions are to be explained; but for present purposes we will treat it as ultimate. (p. 111/8–16)

In order to really understand what Russell’s general theory of complexes comes to, we must somehow cash out this reduction.

What Russell seems to have in mind here are his newest entities, logical forms. All complexes have a logical form, not as additional constituents, but as ‘the way in which the constituents are put together’ (p. 98/23–4). Logical forms come to play a central role in the multiple-relation theory, but independently of that they seem needed to account for what complexes are logically possible in terms of something actual. One proposal is just to say that a description which fails to pick out an actual complex nevertheless describes a possible complex when there is an actual complex whose constituents are pairwise homogeneous with the constituents of the description. That is, the merely possible complex has the same form as an actual complex. For Russell, this will work only if there are logical forms, and so he introduces them into his ontology.

It must be admitted that this interpretation of Russell’s account of logically possible complexes goes beyond what he explicitly says, and does not immediately resolve a dispute between someone who insists that when A is a part of α it is logically possible that α is a part of A and someone, like Russell, who denies this. The same point can be made against Russell’s distinction between thing and concept in Principles: in virtue of what are the propositions expressed by ‘Socrates is human’ and ‘Humanity is possessed by Socrates’ distinct? On my reading, Russell is willing to countenance certain metaphysical brute facts and is not troubled by the demand for a deeper account of these facts. This makes it easier for Russell to solve some of his problems with the multiple-relation theory, but I will argue shortly that one crucial problem remains.

In an effort to further isolate what logical forms really are Russell tentatively identifies them with completely general complexes whose instances we would normally say had that form. For example, the form of A’s being similar to B would be the complex of there existing a relation R, term x, and term y such that $R(x, y)$. Confusingly, the logical form itself has no constituents and so Russell calls it simple. As it is par-

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11 Compare claims against possibles on p. 152.
adoxical, to say the least, that some complexes are simple, Russell calls such logical forms facts, whereas elsewhere in Theory of Knowledge complexes and facts are identified (p. 80/1–3). There are many problems with this identification of logical forms with general facts, but I will pass them by as this identification does not seem to me to be essential to Russell’s account. We will return later to the significance of Russell’s conflation of complexes and facts.

A final epicycle in Russell’s theory is introduced in the course of his discussion of acquaintance with relations (pp. 79–89). Russell argues that pairs of asymmetrical relations like beforeness and afterness are, despite appearances, really single relations, here sequence. Thus, the complex A’s being before B is identical to the complex B’s being after A, even though it is different from B’s being before A. How can we identify the right complexes and maintain the needed distinctions, if there is just one relation here, sequence? Russell’s idea is to introduce new relations which he calls position-in-the-complex relations. Here, in addition to the relation of sequence, we have two position-in-a-sequence-complex relations, which he calls earlier and later. Whenever there is a complex in which sequence relates two terms, exactly one term will stand in the earlier than relation to this complex and exactly one will stand in the later than relation to this complex. So, by metaphysical fiat, we distinguish A’s being before B from B’s being before A by saying that even though both complexes involve the same constituents, A, B, and sequence, in the former case A is earlier, while in the latter B is earlier. A key assumption here is that if A is earlier in α, there is no logically possible complex in which A is earlier than A. That is, A and α are heterogeneous.

Granting the problems we saw about cashing out which complexes are logically possible, we see that Russell offers a completely general account of complexes. It entails that A’s being similar to B and that A’s being a part of α are logically possible complexes, but that similarity, A, B, and α’s being a part of A are not.

3. Understanding

The link between Russell’s general theory of complexes and the multiple-relation theory is simply that propositional attitudes are complexes of a certain kind which involve the judging subject. Almost every commentator assumes that propositional attitudes are explained using new judging, desiring, etc. relations in line with our earlier representation of Othello’s judgement as J(o, d, L, c). There is certainly good textual evi-
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dence for this in Russell's writings before Theory of Knowledge, but by 1913 this is no longer his view. A small change is that understanding is now taken as the most fundamental relation between subjects and entities. A more important shift is that understanding itself is no longer a basic relation. Instead, Othello's understanding that Desdemona loves Cassio will involve a more intricately structured complex than $U(o, d, L, c)$ would suggest.

![Diagram of similarity between A and B understood by S](image)

Figure 1: S understands that A and B are similar (p. 118)

Russell offers us this diagram as part of his solution to what is going on when S understands that A and B are similar (Figure 1). The diagram is quite schematic in the sense that we do not quite know what relations all these arrows are supposed to represent. At a minimum, S must be acquainted with the constituents of the proposition: A, B, the relation of similarity, and $R(x, y)$, the logical form of dual complexes. Beyond this acquaintance, the subject must relate A, B and similarity to the logical form in the right way. In particular, similarity is related to the logical form in a different way than either A or B because similarity is the relation.

Exactly why Russell would prefer this complex to positing understanding as a basic relation is never made clear. One motivation seems unconvincing. This is that, if understanding was a basic relation, then there would have to be many basic understanding relations in order to account for all the different propositions that we can understand. Understanding that $Rab$ would be a different relation than understanding that $Hab$, as the former would require a four-place relation, while the latter involves a five-place relation. Such a concern cannot have
been the reason for Russell's rejection of understanding as a basic relation because his new proposal is open to the same objection. Whatever conglomerate of relations is involved in understanding that Rab will have quite a different structure than understanding that Habc. In fact, Russell will argue that understanding a proposition involving a symmetrical relation like similarity will have a different structure than understanding a proposition involving an asymmetrical relation like larger than. As Russell's newer proposal makes more distinctions than his older approach, this first objection cannot have bothered him.

It is more likely that Russell was responding to an objection related to cases of understanding false propositions. He points out that it is not coherent to maintain that the understanding relation unites, say, Desdemona, loving, and Cassio when, in fact, Desdemona does not love Cassio. If they are not united in a complex in the world, then no amount of understanding can unite them either (p. 116/10–36). This motivation explains what is different in the new theory. Logical forms now enter into the understanding complex as an essential constituent. They are what makes it possible for there to be a complex involving Desdemona, loving, and Cassio along with Othello, when, in fact, Desdemona does not love Cassio.

I want to argue now that if we combine this proposal—how to build up an understanding complex from more basic cognitive relations—with Russell's general theory of complexes, Russell can solve many aspects of what I have called the proposition problem. Recall that the proposition problem is to say what objects, properties, and relations are involved in the understanding, belief, hope, etc. of a given proposition. In line with (PART) Russell needs to give at least the constituents of the understanding complex, as he calls what we might call the mental state of understanding. A second requirement is that he give some indication of how these entities must be related for the understanding to occur at a given time. An especially pernicious aspect of this problem is the direction problem. As Griffin puts it,

It is essential that any theory of judgment be able to distinguish (7) S believes that a precedes b from (8) S believes that b precedes a ... Moreover, ... there is a further form of the direction problem which seems to be unique to Russell's theory, for the theory has also to distinguish both (7) and (8) from such putative judgments as (9) S believes precedes a b (10) S believes b a precedes. (Griffin 1985a, p. 219)

Distinguishing between (7) and (8) is sometimes called the narrow form of the direction problem, while ruling out the possibility of (9) and (10) is called the wide form of the direction problem. (T/F) makes
this problem more urgent as if Russell allowed such cases for judgement, \((T/F)\) would entail that the judgements are either true or false.

Some instances of the proposition problem are easily solved at this point. \(S\)'s understanding that \(A\) and \(B\) are similar is easily distinguished from (1) \(S\)'s understanding that \(D\) and \(B\) are similar, (2) \(Q\)'s understanding that \(A\) and \(B\) are similar, and (3) \(S\)'s understanding that \(A\) and \(B\) are the same colour. (1), (2), and (3) have the same form as Russell's diagram, but with substitutions for \(A\), \(S\), and similarity, respectively. What about the wide direction problem—why can \(S\) not understand that similarity \(A\) \(B\)? Such understandings are ruled out because there is no logically possible complex with the form of Russell's diagram and similarity substituted for \(A\). A description of these constituents in these positions fails to describe a logically possible complex. Russell can solve this kind of direction problem, then, by resting on his general theory of complexes.

This still leaves the narrow version of the direction problem. Note that it does not arise for this case because there is no difference, for Russell, between \(S\)'s understanding that \(A\) is similar to \(B\) and \(S\)'s understanding that \(B\) is similar to \(A\). But it does arise, and Russell does discuss it, for the case where \(S\) understands that \(A\) is before \(B\). Russell's strategy is to adapt his general solution for all such relational complexes to the particular case where the complexes are instances of understanding. If Russell accepts the account of relational complexes that we saw in the last section, it should come as no surprise that he resurrects it to explain what is going on when \(S\) understands that \(A\) is before \(B\). We cannot just replace similarity with the relation of sequence in Russell's diagram because that would run together two cases: (1) \(S\) understands that \(A\) is before \(B\), (2) \(S\) understands that \(B\) is before \(A\). Russell's proposal is simple: 'The proposition "\(a\) is before \(b\)" must be interpreted as meaning "there is a complex in which \(a\) is earlier and \(b\) is later"' (p. 135/13-14). This claim requires the existence of a new kind of complex, that is a molecular complex, due to the presence of 'and'. The existential quantifier also introduces new complications. What Russell is proposing is that \(S\) must be related to \(A\), \(B\), earlier, later, and whatever is needed to handle conjunction and the existential quantifier. There is some evidence that the understanding of molecular propositions would involve new 'molecular' logical forms. We cannot be sure, though, as Russell never wrote Part III of his manuscript in which he was to have dealt with molecular complexes and the understanding of molecular propositions.
Whatever the details, Russell hoped to solve the narrow direction problem by arguing that this sort of further analysis was always possible. Whenever an understanding complex looked ambiguous, that is whenever it appeared permutative, analysis would reveal it to be really non-permutative. S does not really understand that \(A\) is before \(B\), but rather that there is a complex in which \(A\) is earlier and \(B\) is later. While Russell does not fully realize this ambitious program, by positing enough additional position-in-the-complex relations and logical forms, he clearly hoped to solve all aspects of what I have called the proposition problem.

4. Some objections

4.1 The direction problems

With this background in mind, I turn to a consideration of the most influential explanations of why Russell gave up the multiple-relation theory. There is conclusive textual evidence that this was due to an objection or objections raised by Wittgenstein, but no consensus on what the problem was. The interpretative claim that I want to make in this section is that none of the standard objections are problems that Russell had reason to take seriously. Given this, even if the extant remarks by Wittgenstein suggest a particular kind of objection, that objection cannot be the one that Russell was stymied by. Instead, in the next section, we will see an objection that Russell does consider in the text and that he has every reason to take seriously. Finally, in the section after that, we will see how this objection fits with Wittgenstein's known remarks and the broader clash between Russell and Wittgenstein in this period.

Hanks has recently called Griffin's interpretation 'the standard reading' (Hanks 2007, p. 130), so we will begin with it. To distinguish a judgement like \(S\) believes that Socrates is mortal from \(S\) believes that mortality is Socrates, Griffin thinks that Russell must appeal to type theory: 'Russell's defence against making the substitution the wrong way round lies in type restrictions on admissible substitutions. Wittgenstein's point is that by treating "mortality" as a proper name, type restrictions break down' (Griffin 1985a, p. 230). On this reading, the reason that \(f(o, d, L, c)\) is a possible complex, but that \(f(o, L, d, c)\) is not, is that Desdemona and the loving relation are of different logical types. This is unacceptable, however, as the theory of types must be grounded

\[13\] See also Griffin 1985b.
in the multiple-relation theory of judgement. Several different objections have been raised to Griffin’s interpretation, but for our purposes the easiest response is to note the distinction between kinds and types that I introduced when discussing (CAT). The theory of types applies to propositional functions and propositions, and is indeed grounded by the multiple-relation theory. But in articulating the multiple-relation theory, Russell does not need to appeal to entities of different types. Instead, only entities of various kinds drawn from one metaphysical category enter into his general theory of complexes. In restricting the scope of the theory of types I follow Landini, who views the theory of types as a theory that stratifies propositional functions (Landini 1991, p. 63). Whatever they are, it is clear that propositional functions and Russell’s properties and relations should not be identified. Even Bernard Linsky, who disagrees quite dramatically with Landini’s interpretation of Russell’s logic in *Principia Mathematica*, is eager to separate properties and relations from propositional functions (Linsky 1999).

4.2 The unity problem

Two recent discussions of the multiple-relation theory have argued against Griffin’s proposal and offered two distinct alternatives tied to what is sometimes called the problem of the unity of the proposition. The issue is raised but apparently not resolved in *Principles* when Russell admits that he is not able to say in virtue of what the complex A’s differing from B is distinct from an exhaustive list of its constituents: A, difference, B.14 All that he can offer us is that in the complex the relation is actually relating A to B, whereas when that same relation is picked out by listing the constituents, the relation is not relating A to B. Stevens argues that this problem plagued Russell for decades, even until his last sustained philosophical work in the 1940s. Indeed a version of this problem is said to sink the multiple-relation theory. According to Stevens, the problem is

> the failure of the theory to account for the division of propositional content into parts which will reflect and preserve its unity and hence debar nonsensical pseudo-judgements such as ‘this table penholders the book’ or ‘Love desdemonas Cassio’. (Stevens 2005, p. 105)

Stevens’s idea is that when a relation is not a relating relation for a complex, Russell must treat it as on a par with terms that are not relations. As the multiple-relation theory requires invoking relations that are not relating relations, the theory cannot make the required distinctions. For

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14 For an excellent overview of these issues see Hylton 1990, esp. Chs 4 and 5.
example, if Russell allows $U(o, d, L, c)$, then he must also allow $U(o, L, d, c)$, that is, Othello understands that love desdemona Cassio.

Now, on the interpretation of the theory that I have developed, the complexes that Stevens says Russell must countenance are ruled out as logically impossible. No commitments that Russell defends in *Theory of Knowledge* require what Stevens requires. For example, Russell's theory of complexes denies that if there is a complex where $A$ is a part of $a$, then there is a possible complex in which $a$ is a part of $A$. So even things are heterogeneous in some complexes. When it comes to the network of relations involved in the full analysis of an understanding complex, the same restrictions will be in place. We can block Stevens's objection, then, by emphasizing Russell's theory of complexes and the restrictions that it imposes.

Hanks is also impressed by Russell's struggles with the unity of the proposition, but emphasizes a different issue that will turn out to fit much more closely with the problem I will get to in the next section. After criticizing Griffin's proposal and reviewing some remarks by Wittgenstein, Hanks states his interpretation:

I think Wittgenstein's point is that judging that $p$ is always judging that $p$ is true. This means that we can rephrase the question 'What does $A$ judge?' as 'What does $A$ judge to be true?'. And now the answer that $A$ judges that $a$, $b$, and $R$ are true obviously makes no sense. The collection of $a$, $b$, and $R$ is not the sort of thing that can be true or false. Only a proposition can be judged to be true—a collection of items, *even if they are of [the] right number and variety of types*, is not the sort of thing that can be true or false and hence not the sort of thing that can judged. (Hanks 2007, pp. 137–8)

The key move here is the claim that the multiple-relation theory is committed to the thing or things to which the subject is related in judgement being the bearer of truth or falsity. On Hanks's proposal, Russell did not initially recognize this feature of judgement, but that after discussions with Wittgenstein, he came to realize that this was an essential feature of judgement.

The connection to truth and falsity is, I believe, a marked step in the right direction. In saddling Russell with (T/F), I have perhaps gone even further than Hanks in requiring that any theory that Russell would accept must preserve bivalence for judgement. Where I part with Hanks is his assumption that it is the collection involved in the understanding or belief complex that is the bearer of truth or falsity. Instead,

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15 As Stevens ascribes (SUB) to Russell, he perhaps thinks that Russell also adheres to (SUB*): Replacing an entity in any complex with an entity in another complex always yields a complex. I cannot pursue the connections between (SUB) and (SUB*) here except to say that even if Russell maintained (SUB) prior to 1913, he does not believe (SUB*) in 1913.
on my development of the theory, it is the belief complex itself that is true or false. Assuming that the complex is unified by the believing relation, I see no problem with it having properties, including the properties of being true or being false. At least initially, Russell saw no problem either. Hanks's interpretation requires ascribing to Russell a principle which he shows no interest in once he has given up the identity theory: Only what is judged, and not the entire propositional attitude, can be true or false. Once the shift to the multiple-relation theory has been made, Russell does not believe in a distinct sort of entity that is judged and so he must explicitly deny that only what is judged can be true or false. Of course, Hanks is quite right to see Russell adhering to the principle that only things, and not collections or lists of things, can have properties. But, on my reconstruction, Russell finds things, namely the belief complexes, to be the bearers of truth and falsity. Such complexes are just as unified as his earlier propositions were.

There is a worry here, as Russell seems to replace a basic understanding relation with a series of simpler relations between the subject and the constituents of the proposition. For if understanding is no longer a basic relation, then the complex sketched in Figure 1 will not have any unity and so perhaps cannot be the bearer of any properties after all. Russell shows no awareness of this potential problem and sometimes refers to understanding as a relating relation, while in other places seems to identify the understanding complex with a series of simpler relations. On his behalf, we might say that understanding is a relation that obtains if and only if a series of other relations relate appropriate entities. Hanks's objection thus reveals an important metaphysical cost of Russell's proposal. But as long as Russell is willing to pay this price, his multiple-relation theory can escape this objection.

4.3 Logical form
The discussion that has most directly influenced my own presentation is Landini's 'A New Interpretation of Russell's Multiple-Relation Theory of Judgment' (Landini 1991). Landini emphasizes the importance of logical forms for Russell's new version of the multiple-relation theory and how this change allows Russell to overcome various possible objections to his old theory. Also, Landini is one of only two commentators to note the passage about correspondence that I will turn to in the next section (Landini 1991, p. 56, n. 9). But instead of developing this issue,
he focuses more squarely on the logical forms and their peculiar status. If logical forms cannot do the job that Russell needs them to do, but Russell now sees that logical forms are presupposed by the multiple-relation theory, then we can see why Russell would give up the theory.

The clearest statement of what I take Landini’s objection to be is the following:

For Russell, understanding logical form is presupposed in belief. But such a theory [of logical forms] must say what can only be shown. Allowing logical forms as constituents of belief-complexes accomplishes nothing in Wittgenstein’s view. Our understanding (e.g.), of the logical impossibility of a concrete particular occurring as a relating relation, cannot be explained by appeal to our acquaintance with an extra entity, a fact, which is an object before the mind. (Landini 1991, pp. 66–7)

The proposal, with its allusions to Wittgenstein’s account of representation in the *Tractatus*, may very well explain how Wittgenstein came to think of the multiple-relation theory and its failings. It also fits with what we know of Wittgenstein’s objections. Still, nothing that Landini says makes it clear how simply making this sort of point could have deflected Russell from his philosophical path in *Theory of Knowledge*. What is needed is an argument, based on premisses that Russell would have accepted at this time, whose conclusion is that the multiple-relation theory cannot explain what understanding a proposition consists in.

5. The correspondence problem

So far I have argued that Russell has a new version of the multiple-relation theory in *Theory of Knowledge* and that it has the resources to overcome the standard objections that are made to it. Still, even if these kinds of problems were to be solved, the multiple-relation theory is not yet complete. For at least for some propositional attitudes, for example belief, a truth value is involved. When Othello believes that Desdemona loves Cassio, Othello takes it to be the case that Desdemona really does love Cassio. This prompts Russell to try to solve what I am calling the correspondence problem: for any belief complex, what must be the case for this belief to be true and what must be the case for the belief to be false? I contend that by his own lights Russell came to see that the multiple-relation theory could not solve this correspondence problem.18

Intuitively, we can see why an account might solve what I earlier called the proposition problem and yet fail to handle correspondence.

18 See Russell 1906, p. 46, n. where this problem is first noted.
To see the difference, let us reconsider the four beliefs that Griffin said must be distinguished to solve the direction problem:

(7) S believes that \( a \) precedes \( b \)
(8) S believes that \( b \) precedes \( a \)
(9) S believes \( a \) precedes \( b \)
(10) S believes \( b \) precedes \( a \)

This is the problem of saying in virtue of what the complex picked out by each of (7)–(10) is different from the others, or else why it is logically impossible for there to be such a complex. What we will see shortly is that even if we grant Russell the metaphysical machinery to make these necessary distinctions, he is still unable to show that for each logically possible belief complex \( B \) there is a complex whose existence is necessary and sufficient for \( B \)'s truth. For example, for (7), Russell realizes that there is no acceptable description of a complex whose existence is necessary and sufficient for (7)'s truth. So, even if the internal constitution of the propositional attitudes are adequately described, we might still fail to specify the essential relations of these attitudes, that is the relations that are necessary and sufficient for truth.

Concerns about the truth or falsity of belief are not entirely absent from Griffin’s or Stevens’s discussions, but the following passage from Stevens presents what I take to be a typical conflation of the proposition and correspondence problems: ‘Russell’s problem, whether in its 1903 incarnation or its 1913 one, had been to explain how propositional constituents gel together to yield propositional content’ (Stevens 2006, p. 112, my emphasis). Recall that Stevens’s objection to the multiple-relation theory concerns how the constituents of a propositional attitude form a unity. But here it is clear that he also has issues of content in mind, which in Russell’s case are cashed out in terms of the truth conditions of the propositional attitude. So, we can read Stevens here as claiming that the problem with the multiple-relation theory is that it does not provide complexes with the right truth-conditions because it does not provide complexes at all. What I will argue presently is that Russell saw that the multiple-relation theory does not provide complexes with the right truth-conditions even though it provides sufficiently distinguished complexes.

This issue might seem close to the problem we saw Hanks raise in the last section. Still, there are important differences between the correspondence problem and Hanks’s objection. Hanks makes Wittgenstein
force Russell to realize that the entities constituting what is believed are neither true nor false. I replied that Russell tries to make the belief complex itself the bearer of truth and falsity. Still, considering the correspondence problem will reveal that Russell had no good way to sort his belief complexes into those that were true and those that were false. So, no definition of truth in terms of correspondence is forthcoming.

Here I see three premisses which together sink the theory. First, Russell never wavers from (PART). Second, the belief complex must be of the same form as the understanding complex delivered by his earlier analysis (p. 137/14–19). Finally, he insists that the correspondence relation must be a function defined on the genuine constituents of the belief complex: ‘The belief is true when the objects are related as the belief asserts that they are. Thus the belief is true when there is a certain complex which must be a definable function of the belief, and which we shall call the corresponding complex, or the corresponding fact. Our problem, therefore, is to define the correspondence’ (p. 144/15–20). Russell does not say why it ‘must be’ a function, but we can see it as a natural consequence of (T/F) and (PART). Being either true or false is an essential feature of the belief complex and so one disjunct of this disjunctive property must attach to it automatically.

If the belief complex is atomic and non-permutative, then the correspondence problem is easily solved. This is because there is only one logically possible complex for the constituents of the propositional attitude. So, the belief complex will be true if and only if the description of a complex listing the constituents is satisfied. For example, S’s belief that A and B are similar will be true if and only if there is a complex with constituents A, B, and similarity and no other constituents.

Problems arise when Russell tackles the truth-conditions of the belief complexes in prima facie permutative cases like S’s belief that A is before B. Russell states his goal clearly: ‘When several complexes can be formed of the same constituents, to find associated complexes unambiguously determined by their constituents’ (p. 145/10–11). Russell goes on to rehearse his account of such complexes and their position-in-the-complex relations. Here we already know that the belief complex can be more perspicuously described as S’s belief that there is a complex in which A is earlier and B is later. So, unsurprisingly, the truth-conditions of this belief complex will also be given in terms of these position relations. If there is a complex such that A is earlier in it and B is later in it, then the belief complex is true. Otherwise, the belief complex is false.

This proposal is then generalized to any n-termed relation R. Such a relation requires the existence of n position-in-an-R-complex relations
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Then, S’s belief that \( R x_1 \ldots, x_n \) will be more perspicuously written as the belief that there is a complex \( \gamma \) such that \( x_1 C_1 \gamma, \ldots, x_n C_n \gamma \). And the belief will be true if and only if there is such a complex. Is the correspondence problem so easily solved?

Russell returns three times over the next nine pages to what is essentially the same problem with this proposal. The problem is that the ultimate constituents of the belief complex cannot be univocally mapped to a logically possible complex. In our simple example, \( S, A, B, \) earlier, later, and whatever is needed to handle conjunction and existential quantification are the ultimate constituents. \( S \) is of course not supposed to be relevant. That leaves two logically possible complexes: the complex in which \( A \) is earlier and \( B \) is later, and the complex in which \( B \) is earlier and \( A \) is later. So we cannot give a specific enough description using the ultimate constituents of the belief complex. What else could we use? It might seem like we could appeal to the atomic constituents of this belief complex: \( A \)’s being earlier in \( \gamma \) and \( B \)’s being later in \( \gamma \). But for Russell this threatens to reintroduce false propositions as freestanding entities. For suppose that \( A \) is not before \( B \). Then there will not be a complex in which \( A \) is earlier, even when \( S \) believes that \( A \) is before \( B \). This tells Russell that he cannot appeal to such atomic constituents, but he is left with no other options.

Let’s turn to Russell’s explicit remarks on this issue. After discussing the correspondence problem and his solution for the sequence case, Russell notes an objection:

> It may be said, of course, that ‘\( A \) is earlier in \( \gamma \) and \( B \) is later in \( \gamma’ \) is composed of the same constituents as ‘\( A \) is later in \( \gamma \) and \( B \) is earlier in \( \gamma’ \). But these are both molecular complexes, and the atomic complexes which enter into them are different; the identity of constituents only appears when we carry our analysis further, to the constituents of the atomic complexes. And this remoter identity of constituents does not raise the problems with which we are at present concerned. (p. 145/30–6)

Only two pages later, after giving the general form of his proposal, Russell again fails to recognize the depth of the problem: ‘Whether any difficulties arise from the fact that the molecular complex is still permutative with respect to the constituents of its atomic constituents, is a question which must be left until we come to deal with molecular thought. But it seems fairly evident that no difficulties can arise from this fact’ (p. 147/22–6). That is, with respect to its ultimate constituents, more than one complex is logically possible. So a list of these constituents cannot determine a complex. But what else can we have as inputs into our function without countenancing false propositions?
The third and most extensive treatment of the problem comes in response to what Russell presents as the first objection to his definition of truth. This is that 'the correspondence between belief and fact is arbitrary on our account' (p. 153/36–7). In the details of his discussion, though, Russell moves quickly from the worry about arbitrariness to an alleged gap between the belief complex and its truth-conditions.

Where permutative complexes are concerned, our process of obtaining associated non-permutative complexes was rather elaborate, and no doubt open to objection. One special objection is that, [1] in order to regard the associated complex as non-permutative, we have to regard its atomic constituents, \( x_1C_1\gamma, x_2C_2\gamma, \) etc., as really constituents, and what is more, [2] we have to regard the corresponding propositions as constituents of the propositions 'there is a complex \( \gamma \) in which \( x_1C_1\gamma, x_2C_2\gamma, \) etc.' This seems to demand a mode of analyzing molecular propositions which requires the admission that they may contain false atomic propositions as constituents, and therefore to demand the admission of false propositions in an objective sense. This is a real difficulty, but as it belongs to the theory of molecular propositions we will not consider it further at present. (p. 154/16–28, my numbering)

Thus in a few pages what was evidently not a problem has become a 'real difficulty'. [1] looks like a claim about the complexes themselves and what their metaphysical structure must be for correspondence to work this way. [2] focuses in more directly on the belief complex itself, and what its structure must look like if the correspondence function is to be univocal. Without assuming that \( x_1C_1\gamma \) obtains in the belief complex, we lose the mapping from the belief complex to a single non-belief complex. That is, we require that \( x_1 \) really stand in the \( C_1 \) relation to \( \gamma \) in the belief, even if, in the world, \( x_1 \) does not stand in the \( C_1 \) relation to any \( \gamma \).

The dilemma, then, is clear: Russell either must stick to ultimate constituents and so fail to define correspondence or else introduce additional constituents in the form of false propositions. I claim this objection, raised only 21 pages before the manuscript ends, is the decisive one.

It is hard to understand what Russell planned to appeal to here to solve this problem in his discussion of molecular complexes. One suggestion is for Russell to introduce molecular logical forms like the logi-

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19 In a response to Landini, Hochberg notes these passages and says 'Russell appears to have no solution ... Such problems may have contributed to Russell's rejecting the later parts of the manuscript' (Hochberg 2001, p. 87). However, in a footnote on the same page Hochberg also claims, noting the p. 147 passage, that 'Russell seems to have thought no problem arose in such a case'. Hochberg does not link these issues to Wittgenstein's criticisms or Russell's later views.

20 See p. 153/6–12 for yet another point where Russell alludes to this issue.
cal forms that we have already seen for atomic complexes. At one point Russell explicitly introduces such things: 'a molecular form is not even the form of any actual particular: no particular, however complex, has the form “this or that”, or the form “not-this”' (p. 132/18–20).21 One way out would then seem to be to allow molecular forms and to use them to define the correspondence relation. Suppose, for example, that we had a conjunctive form \( p \AND q \). Then S's belief that A is earlier in G and B is later in G, for some particular complex G, would be true if and only if there was a molecular complex in which A, earlier, and G formed an atomic complex in the first slot and B, later, and G formed an atomic complex in the second slot. We appeal to the structure offered by the molecular form in order to group A with earlier and B with later.

This still leaves it open how Russell was planning on handling the existential quantification needed to make sense of the belief that there is some complex \( \gamma \) with the right constituents. One possibility, suggested by the introduction to Principia Mathematica (Ch. 2, Sect. III), is that there are different orders of propositions, and that the truth of higher-order propositions can only be defined using the truth or falsity of various lower-order propositions. In our case, the belief that there is a complex \( \gamma \) in which \( x_1C_1\gamma \AND x_2C_2\gamma \) would be true if and only if, somewhere in the world, there was a complex that was true and of the form S believes that \( x_1C_1G \AND x_2C_2G \). But how can we pick out this complex so that we distinguish it from a complex of the form S believes that \( x_2C_1G \AND x_1C_2G \)? Both share the same form and, at the level of their ultimate constituents, reduce to the same entities. Unless we bring in atomic complexes at some point, no uniquely corresponding complex can be found.

A final, desperate, thought would be to invoke existential complexes with their special forms.23 In this case, \( (\exists \gamma)(-\gamma \AND +\gamma) \) would be the relevant form. While this initially sounds reasonable enough, it faces exactly the same problem that we saw earlier. For how are we to stipulate that A and earlier go in the first slot except by introducing new position-in-the-molecular-complex relations? Call our intended general molecular complex \( \Omega \). Then we could say that A and earlier must

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21 Compare p. 99/22–7. Also, it is hard to see how Russell can allow molecular complexes at p. 80/4–10 without molecular logical forms. Landini (1994, p. 52) argues, based on a broader interpretation of Russell's philosophy of logic, that there are no molecular complexes and that Russell's remarks about molecular forms should not be read this way.

22 This fits with the provisional account of propositions suggested at p. 115/29.

23 There is no evidence of this in Russell prior to the general facts of 'The Philosophy of Logical Atomism', but given this later development, it is worth seeing why it does not help with this specific problem.
be first in $\Omega$, while $B$ and later are second in $\Omega$. This gives us more constituents, but still when we get down to the ultimate constituents we will not be able to distinguish the two relevant complexes: $A$'s being before $B$ and $B$'s being before $A$. Again the problem is that our intended complex $\Omega$ in which $A$ is earlier has the same ultimate constituents as the unintended $\Omega'$ in which $B$ is earlier. What we need is for $A$ to really be earlier in $\gamma$, for such a complex would allow us to appeal to this complex in specifying the correspondence. That is, we need a way to make $A$'s being earlier in $\gamma$ a genuine constituent of the belief complex even when $A$ is not earlier in $\gamma$ (i.e. $A$ is not before $B$).

6. Wittgenstein

While there can be no doubt that Wittgenstein's objections were the cause of Russell's abandonment of the Theory of Knowledge manuscript and the multiple-relation theory, the nature of these objections is not revealed in any of the materials extant from 1913 or later. The best that we can hope for, then, is a reconstruction of the concerns that Wittgenstein raised that is consistent with the evidence that we now have. I want to argue in this section that all surviving documents are consistent with Wittgenstein raising a challenge related to truth and the correspondence problem. This clashes with every other interpretation that I know of. Others see Wittgenstein focusing on some kind of proposition problem, either having to do with the unity of the proposition or the direction problem or both. I do not deny that these interpretations get some support from what we know about what Wittgenstein said. Still, if what I have said so far in this paper is at all accurate, then Russell had no reason to accept the criticisms indicated by these interpretations. That he should have been worried about correspondence, and was in fact worried, was the main claim of the last section.

First, I present the timeline of the Wittgenstein–Russell interactions around the Theory of Knowledge manuscript, and some later documentary evidence. Then, I turn to the lessons that Russell seems to have learned from these interactions. I argue that they are consistent with concerns about defining truth.

6.1 A timeline

Wittgenstein's first criticisms were delivered in person on May 20th, as Russell was working on the first part of Theory of Knowledge. They were directed, according to Russell, at 'the theory of judgment which I used to hold' (p. xix). By May 24th, Russell had begun working on Part II of
the manuscript dealing with atomic propositional thought. Then, on May 26th Wittgenstein returned with more criticisms: he said that what Russell showed him was 'all wrong, not realizing the difficulties—that he had tried my view and knew it wouldn't work' (p. xix). By May 31st, Russell convinced himself that he could 'circumvent Wittgenstein's problems'. Significantly, on the next day, June 1st, Russell completed chapter five 'On Truth and Falsehood', which contains the objection we have just reviewed. It appears on p. 154 of a chapter than runs from p. 144 to p. 155. This strongly suggests that Russell wrote this passage on June 1st. On June 1st, Russell wrote to Morrell saying 'I have only superficially and by an act of will got over Wittgenstein's attack—it has made the work a task rather than a joy' (quoted at Griffin 1985b, p. 143). At this point Wittgenstein was away and Russell continued to work on the manuscript, completing Part II on June 6th (Griffin 1992, p. 447).

Shortly before Russell was to meet with Wittgenstein and his mother on June 18th, Wittgenstein wrote a letter to Russell which includes this objection:

I can now express my objection to your theory of judgment exactly: I believe it is obvious that, from the proposition 'A judges that (say) a is in the Relation R to b', if correctly analysed, the proposition 'aRb v. ¬aRb' must follow directly without the use of any other premiss. This condition is not fulfilled in your theory. (Wittgenstein 1974, [R. 12])

By June 19th Russell admitted to Morrell that Wittgenstein’s objections were decisive: ‘it makes a large part of the book I meant to write impossible for years to come probably’ (p. xx). Presumably in response to a similar admission in a letter to Wittgenstein (now in Austria), Wittgenstein replied on June 22nd: ‘I am very sorry to hear that my objection to your theory of judgment paralyses you. I think it can only be removed by a correct theory of propositions’ (Wittgenstein 1974, [R. 13]).

The first point to make is that ‘the theory of judgment which I used to hold’ might very well be the theory which takes judgement as a basic relation. Russell dispenses with such a relation in the manuscript and, among other things, this gives him more options for handling the definition of correspondence for false judgements. Russell then believes that he can ‘circumvent Wittgenstein’s problems’ as he starts to write the chapter on truth in Part II. This expression of optimism fits with the early versions of the objection about correspondence, which appear in the manuscript on p. 145 and p. 147. But misgivings appear in the material presumably written the next day, on June 1st, when Russell gives his most considered discussion of the correspondence problem and grants its significance. In the remaining five days that he worked on
the manuscript, we saw that Russell had growing misgivings about the project, and this fits with my proposal that he came to realize that there was no hope of settling the correspondence issue in Part III. Finally, the June 18th letter says that it contains a more exact version of the concerns. On my reading, when Wittgenstein writes that 'A judges that a is in the Rel[ation] R to b' must entail 'aRb. V. ¬aRb', what he is saying is that judgement presupposes that the judgement is either true or false. Given that Russell is not able to define correspondence relations for his belief complexes, this objection is conclusive if he retains (T/F).

6.2 Complexes vs. facts
We saw earlier how Russell’s metaphysics led him to conflate complex things or complexes with facts. After 1913, Russell is much more careful about this and often ascribes the insight to Wittgenstein. I want to present this shift as a natural response to the difficulties tied to defining correspondence. In effect, Wittgenstein convinced Russell that if he was to maintain (T/F), then he must give up (CAT) in favour of a two-category metaphysics of things and facts. It appears that Russell still thought of maintaining (PART) and some kind of multiple-relation theory. But when he saw a way out that would sacrifice (PART), but preserve (T/F), he took it. This is what happened in Russell’s work on belief after the ‘Philosophy of Logical Atomism’ lectures (Russell 1986, pp. 155–244).

The connection between a rigid thing/fact distinction and the correspondence problem is fairly direct. Russell makes this point in the first lecture, where propositions are now complex symbols:

A man believes that Socrates is dead. What he believes is a proposition on the face of it, and for formal purposes it is convenient to take the proposition as the essential thing having the duality of truth and falsehood. It is very important to realize such things, for instance, as that propositions are not names for facts. It is quite obvious as soon as it is pointed out to you, but as a matter of fact I never had realized it until it was pointed out to me by a former pupil of mine, Wittgenstein. It is perfectly evident as soon as you think of it, that a proposition is not a name for a fact, from the mere circumstance that there are two propositions corresponding to each fact. Suppose it is a fact that Socrates is dead. You have two propositions: ‘Socrates is dead’ and ‘Socrates is not dead’. And those two propositions correspond to the same fact, there is one fact in the world which makes one true and one false. This is not accidental, and illustrates how the relation of proposition to fact is a totally different one from the relation of name to the thing named. (Russell 1986, p. 167/27–41)
The ‘duality’ of propositions requires that they represent a different kind of entity than what names stand for, as names simply either stand for a thing or are meaningless.

(CAT) is now replaced by a principle that puts things in a single category and facts in another. The test is again whether or not an entity is the logical subject of some proposition. Given that ‘You can never put the sort of thing that makes a proposition to be true or false [i.e. a fact] in the position of a logical subject’ (Russell 1986, p. 168/22–4), facts are in a different category than things. This leaves the category of complex things in a somewhat unstable position. It is tempting to think that Russell shifted to a metaphysics in which all things are simple, just as Wittgenstein presents his metaphysics of objects in the *Tractatus*. A text in favour of this interpretation is the claim, in the last lecture, that ‘you can get down in theory, if not in practice, to ultimate simples, out of which the world is built, and that those simples have a kind of reality not belonging to anything else’ (Russell 1986, p. 234/33–5). In response to a question at the end of lecture II, though, Russell suggests that even if this is correct, it is not guaranteed: ‘I do myself think that complexes—I do not like to talk of complexes—but that facts are composed of simples, but I admit that that is a difficult argument, and it might be that analysis could go on forever’ (Russell 1986, p. 180/7–10). In addition to showing Russell’s hesitation about simples, this passage supports my suggestion that Russell is now aware of the dangers of assimilating complex things to facts.²⁴

With this background, we can make better sense of the relevant parts of the ‘Notes on Logic’ that Wittgenstein dictated in Russell’s presence in September of 1913. There Wittgenstein claims that ‘Every right theory of judgment must make it impossible to judge that this table penholders the book. Russell’s theory does not satisfy this requirement’ (Wittgenstein 1979, p. 103). How does this remark fit with my reconstruction? Rather than read it as involving the direction problem or the unity problem, I suggest that it is simply Wittgenstein’s way of insisting on (T/F) and the need to reject (CAT). This sort of nonsensical judgement must be ruled out because that is the only plausible way to guarantee that every judgement is either true or false. Other alternatives run afoul of other principles that Russell is unwilling to give up. For example, stipulating that all nonsensical judgements are false entails that the judgement expressed by ‘this table penholders the book or this table does not penholder the book’ is false, thus undermining the unrestricted scope of the law of excluded middle. As a result, it came to seem

²⁴ Compare p. 80/1–3.
for Wittgenstein and then also for Russell that making the duality or bipolarity of propositions the centre of an account of judgement was the only way to spell out how judgements manage to be true or false. This suggestion fits with Wittgenstein's considered verdict on the issue in *Tractatus*, 5.5422: 'The correct explanation of the form of the proposition, "A makes the judgment p", must show that it is impossible for a judgment to be a piece of nonsense. (Russell's theory does not satisfy this requirement.)'

Russell's low point on such questions is reflected in the well-known May 1916 letter to Morrell in which Russell presents Wittgenstein's criticisms as some kind of existential crisis:

His [Wittgenstein's] criticism, though I don't think you realized it at the time, was an event of first-rate importance in my life, and affected everything I have done since. I saw he was right, and I saw that I could not hope ever again to do fundamental work in philosophy. My impulse was shattered, like a wave dashed to pieces against a breakwater ... Wittgenstein persuaded me that what wanted doing in logic was too difficult for me. (Russell 1986, pp. xix–xx)²⁵

It is hard to square this statement with the work in philosophy that Russell did between June 1913 and May 1916. Whatever Russell may have been thinking in 1916, by 1917 there is evidence that he had regained his philosophical composure. A hint of this comes in a footnote added to the 1917 version of 'Knowledge by Acquaintance and Knowledge by Description'. Appended to the claim that there is a supposing relation that obtains between the subject and the constituents of a proposition, he adds

I have been persuaded by Mr. Wittgenstein that this theory is somewhat unduly simple, but the modification which I believe it to require does not affect the above argument. (Russell 1992, p. 154/36–8)

One way to make sense of this 'unduly simple' remark is that Russell still hoped to resolve the correspondence problem by imposing a distinction between things and facts.²⁶

The last discussion of belief that Russell offers before giving up (PART) occurs in the 'Philosophy of Logical Atomism'. Two important points are noteworthy. As we saw, Russell insists that 'The essence of a proposition is that it can correspond in two ways with a fact, in what one may call the true way or the false way' (Russell 1986, p. 185/4–6). Propositions, though, are now thought of as complex symbols. When it


²⁶See 'On Props' (Appendix B.1) for a sketchy attempt along these lines that may date to 1917. A discussion of the interpretation of this manuscript is beyond the scope of this paper.
comes to the facts that propositions that attribute beliefs represent, we get the second major shift in Russell’s philosophy: the introduction of two-verb facts. The proposition ‘Othello believes that Desdemona loves Cassio’ involves a new kind of fact in addition to the atomic and general facts associated with other kinds of propositions like ‘Desdemona loves Cassio’ or ‘Desdemona loves everyone’. In the belief case ‘You will perceive that it is not only the proposition that has the two verbs, but also the fact, which is expressed by the proposition, has two constituents corresponding to verbs’ (Russell 1986, p. 192/20–2). This has some puzzling consequences. If beliefs involve two-verb facts that are different in kind from atomic facts, then we cannot give a spatial diagram of the logical form of such facts. This is because all spatial facts are atomic (Carey 2003).

Whatever the demerits of this proposal, it does seem clear that it solves the problems about correspondence that I have argued bedevilled the Theory of Knowledge approach. For in introducing two-verb facts, Russell has found a way to have atomic complexes like A’s being earlier in γ even when A is not earlier in γ. This is now noted as a puzzling fact about belief, rather than a problem for Russell to overcome:

Suppose I take ‘A believes that B loves C.’ ‘Othello believes that Desdemona loves Cassio.’ There you have a false belief. You have this odd state of affairs that the verb ‘loves’ occurs in that proposition and seems to occur as relating Desdemona to Cassio whereas in fact it does not do so, but yet it does occur as a verb, it does occur in the sort of way that a verb should do. I mean that when A believes that B loves C, you have to have a verb in the place where ‘loves’ occurs. You cannot put a substantive in its place. Therefore it is clear that the subordinate verb (i.e. the verb other than believing) is functioning as a verb, and seems to be relating two terms, but as a matter of fact does not when a judgment happens to be false. That is what constitutes the puzzle about the nature of belief. (Russell 1986, p. 198/9–20)

Crucially, Russell adds a general lesson about the difficulties that arise from failing to attend to this problem. I take him here to be referring back to the correspondence problem that I have emphasized:

You will notice that whenever one gets to really close quarters with the theory of error one has the puzzle of how to deal with error without assuming the existence of the non-existent. I mean that every theory of error sooner or later wrecks itself by assuming the existence of the non-existent. (Russell 1986, p. 198/20–4)

In the Theory of Knowledge we saw that Russell’s account of correspondence for false belief did eventually require him to posit complexes that were non-existent, and that there is every reason to think Russell was
aware of this problem. By now countenancing two-verb facts, Russell is able to trivially define correspondence. Desdemona’s loving Cassio occurs as a unit in the two-verb fact whether or not Desdemona really loves Cassio. The belief will then be true if Desdemona really does love Cassio.

7. Conclusion

I would not claim that this new proposal is satisfactory and I wonder if in the end it can be made coherent. I for one am not happy introducing a new kind of fact in which a part of it will be Desdemona loving Cassio, but where it must still be an open question whether Desdemona really loves Cassio. The move is actually quite reminiscent of Russell’s first account of false propositions. At that stage, falsity was a simple property that attached to some propositions and not to others. Now, the false proposition is a fact that occurs, as a fact, in a two-verb fact like a belief fact. Exactly how occurring in this sort of context can be consistent with the fact not occurring outside of the belief fact is never explained.

Russell also seems quite unhappy with the two-verb fact proposal, and this helps to make sense of where Russell was soon to turn in his quest for an adequate analysis of belief. One of the three premises that we saw got Russell into this trouble was (PART): the objects, properties, and relations themselves are part of the belief complex, or now the two-verb fact. This is decisively rejected in the 1919 paper ‘On Propositions’ in favour of propositions as images standing in relations (Russell 1986, pp. 276–306). In that paper a central question is ‘the manner in which a proposition refers to the fact that makes it true or false’ (Russell 1986, p. 281/36–7), and one of the few things that Russell does is define truth-conditions for some simple examples of his new image propositions (Sect. IV). It is only at this point that he has finally started to overcome the problems about truth that we have seen in the Theory of Knowledge. But by then we no longer have a multiple-relation theory because we no longer have (PART).

The route from ‘The Philosophy of Logical Atomism’ to ‘On Propositions’ is not a direct one, but hints of the shift to come appear in the lectures when Russell discusses alternatives to the two-verb fact approach to belief. His basic idea is that, despite appearances, the two-verb facts considered need not be ultimate facts after all. Perhaps facts of a more ordinary sort could come together to generate a belief fact. After referring to the ‘behaviourism’ of James and Dewey’s anal-
ysis of belief, Russell notes that ‘it is a subject belonging to psychology’ (Russell 1986, p. 193/34) to determine whether or not beliefs isolated from behaviour ever occur. This makes it clear why Russell would turn to psychology after completing these lectures. He clearly hoped that psychology would reveal how to get by without two-verb facts. In prison, from May until September of 1918, Russell read psychology with an eye towards solving his problem about belief. Behaviourism did not prove adequate, but a psychological theory involving images did. The first published results of this psychological turn was the paper ‘On Propositions’, which begins ‘A proposition may be defined as: What we believe when we believe truly or falsely’ (Russell 1986, p. 278/1–2).

We see, then, that the principle behind the rejection of the multiple-relation theory is (T/F), the idea that judgements are essentially true or false, or what Wittgenstein would call the bipolarity of the proposition. When Russell saw that he could not preserve all three of his preferred principles (PART), (T/F), and (CAT), he first blamed (CAT) and adopted a two category metaphysics of things and facts. Still, this turned out not to be consistent with a plausible account of belief facts, and so finally Russell abandons even (PART). (PART) had been central to his rejection of idealism, and when Russell replaces it with a conception of propositions involving images it can seem a bizarre and unmotivated shift. However, once we see the pressures relating to the correspondence problem, we can better appreciate Russell’s reasons, even if we do not share them. Shifting to images made new problems for Russell and a particularly urgent one was how to specify what thing an image is an image of (Pincock 2006). Trying to solve this problem takes up much of the next phase of Russell’s philosophical development.28

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27 Thompson 1975, p. 18, Russell 1986, p. 249, and appendix III.

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Rifssell's Last (and Best) Multiple-Relation Theory of Judgement


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